Symbol Error Probability Analysis of MIMO-Relay Multiplexing Scheme in IEEE 802.16j Systems

*Assad Akhlaq, *Ahmed Iyanda Sulyman, Senior Member, IEEE, +Hossam Hassanein, Senior Member, IEEE, *Abdulhameed Alsanie, and **Saleh Alshebeili

*Department of Electrical Engineering, King Saud University, Riyadh, Saudi Arabia
+School of Computing, Queen's University, Kingston, ON, Canada K7L 3N6.
**Department of Electrical Engineering, KACST Technology Innovation Center RFTONICS, King Saud University, Riyadh, Saudi Arabia
aakhlaq@ksu.edu.sa, asulyman@ksu.edu.sa, hossam@cs.queensu.ca

***Abstract*— This paper presents the symbol error probability (SEP) analysis of MIMO-relay multiplexing configuration for IEEE 802.16j systems. The availability of multiple MIMO relay paths in IEEE 802.16j motivates two relay configurations: i) Relay Diversity and ii) Relay Multiplexing. Existing works have focused on the relay diversity methods. In this paper, we explore the relay multiplexing alternative where relay stations (RSs) are deployed to act as independent data pipes for transmitting multiple independent data streams from the Base Station (BS) to the Mobile Station (MS). We derive analytical expressions for the SEP performance of the proposed scheme, and also complement the analysis with simulations. It is observed from our SEP results that MIMO-relay multiplexing approach is practicable, when the number of antennas at the BS and MS are equal or greater than the total number of antennas on all RSs. In such scenarios, parallel relaying of independent data streams via different RSs offers acceptable SEP performance while boosting the capacity of the system linearly with the number of parallel RSs.**

Keywords-MIMO-relay multiplexing, Decode and forward, Maximum likelihood detection (MLD), Relay access interference.

I. Introduction

Since the introduction of MIMO systems, a lot of research works have been conducted on the application of MIMO systems in different scenarios. Among the many available detection techniques for MIMO multiplexing systems, maximum likelihood detection (MLD) offers optimal performance results. But the exact symbol error probability (SEP) analysis for MIMO multiplexing with maximum likelihood (ML) detection was not available in the literature until 2009 when the authors in [1] provided a very accurate SEP analysis for arbitrary MIMO multiplexing systems. MIMO multiplexing on one hand provides high data rate by multiplexing independent data streams onto multiple antennas. On the other hand, relaying of information from source to destination using multiple RSs, offers additional multiplexing paths that can enhance the data rate provided by MIMO multiplexing systems further. A combination of these two techniques is therefore somewhat indispensable in broadband 4G networks.

Several authors have studied the possibilities of relaying data from the BS to the MS using multiple RSs in a two hop scenario. The performance analysis of different dual-hop systems for various channels is studied in [2]-[6]. All these works, and several others in the literature, however studied the relay diversity method, where the relayed data from all the RSs at any time instant belong to the same sub-stream coded or uncoded. In [7], we provide the capacity analysis of the relay-multiplexing alternative, where multiple RSs are employed for parallel relaying of independent data streams from the same source to the same destination in an IEEE 802.16j setup. In IEEE 802.16j setup, the transmission from BS to MS is carried out in two phases: phase 1 involves data transfer from BS to RSs, while phase 2 involves data relaying from RSs to the MS as shown in Fig. 1. Therefore we propose that using the multiple RSs between the BS and the MS to transmit independent data would help increase the capacity of the IEEE 802.16j system. However the error rate performance for this system needs be examined first before such a conclusion can be drawn.

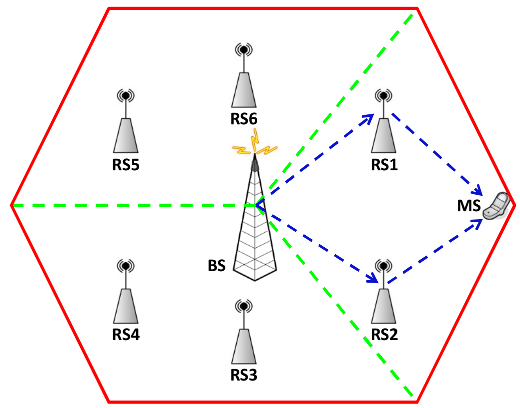

Figure 1: IEEE 802.16 Configuration.

In this paper, we provide the SEP analysis for MIMO-relay multiplexing configuration in IEEE 802.16j system, where each RS forwards an independent sub-stream to the MS. To achieve this objective, a transmit preprocessing technique is applied at the BS to partition the channel between BS and K parallel RSs into K parallel MIMO links for multiple relay-access. This way, the transmission of independent data streams from BS to RSs can be treated as multiuser MIMO downlink system, while the transmission of independent data streams from RSs to the MS can be treated as conventional MIMO-multiplexing system. Although each of these schemes is separately available in the literature, to the best of our knowledge, no one has provided the combination of these techniques that can be used to analyze MIMO-relay multiplexing systems for IEEE 802.16j system as provided in this paper. In this paper, we thus develop analytical tool to investigate the SEP performance of two-hop MIMO-relay multiplexing systems, and based on our results we conclude on the practicability of this technique in IEEE 802.16j systems.

II. Performance Analysis of MIMO Multiplexing Relaying System with MLD

In this section, we highlight a recent result in [1] for the SEP of $N_{t,BS} \times N_{r,MS}$ single-hop MIMO multiplexing system and later extended this result to the $N_{t,BS} \times N_{r,RS} \times N_{r,MS}$ MIMO multiplexing relaying system.

A. *One-hop MIMO Multiplexing System*

Consider an $N_{t,BS} \times N_{r,MS}$ MIMO multiplexing system with $N_{t,BS}$ transmit antennas at the BS and $N_{r,MS}$ receive antennas at the MS. The BS transmits $N_{t,BS}$-length data symbols $\mathbf{x} = [x_1 \quad x_2 \quad \cdots \quad x_{N_{t,BS}}]^T$ towards the MS. The baseband received signal vector at the MS is given by

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n} \tag{1}$$

where $\mathbf{y} = [y_1 \quad \cdots \quad y_{N_{r,MS}}]^T$ is an $N_{r,MS} \times 1$ vector with y_j as the received signal at the j^{th} antenna of the MS. $\mathbf{n} = [n_1 \quad \cdots \quad n_{N_{r,MS}}]^T$ is $N_{r,MS} \times 1$ additive white Gaussian noise (AWGN) vector, each element being independent with zero mean and variance σ_n^2. $\mathbf{H}$ is the channel matrix whose $(j,i)^{th}$ element $h_{j,i}$ is assumed to be an i.i.d complex Gaussian variable with zero mean and unit variance. Let χ denotes the constellation of the modulation scheme employed. It is assumed that all the symbols in the constellation have equal probability. The ML detection, in the presence of Gaussian distributed noise, can be written as

$$\tilde{\mathbf{x}} = \arg\min_{\mathbf{x}\in\chi} \sum_{j=1}^{N_{r,MS}} | y_j - \sum_{i=1}^{N_{t,BS}} h_{j,i} x_i |^2 \tag{2}$$

where $\tilde{\mathbf{x}}$ represents the decision vector for $\mathbf{x}$. Recently the authors in [1] presented an accurate expression for the SEP of MIMO-MLD in Eq. (2), which was previously unavailable. A highlight of their analytical approach is as follows. Let x_i be the transmitted signal from the i^{th} antenna and $\bar{\mathbf{x}}_i = \{x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_{N_{t,BS}}\}$ be the transmitted signals from all other antennas. The SEP of x_i can be written in terms of SEPs conditioned on a set of error events corresponding to $\bar{\mathbf{x}}_i$ and the probabilities of those error events as [1, Eq. (5)]

$$\begin{aligned} P(\tilde{x}_i \neq x_i) &= P(\tilde{x}_i \neq x_i \mid err(\varphi)) P(err(\varphi)) \\ &+ \sum_{\substack{k=1,\\k\neq i}}^{N_{t,BS}} P(\tilde{x}_i \neq x_i \mid err(k)) P(err(k)) + \\ &\sum_{\substack{l=1,\\l\neq i}}^{N_{t,BS}} \sum_{\substack{m=l+1,\\m\neq i}}^{N_{t,BS}} P(\tilde{x}_i \neq x_i \mid err(l,m)) P(err(l,m)) \\ &\cdots + P(\tilde{x}_i \neq x_i \mid err(\Omega)) P(err(\Omega)) \end{aligned} \tag{3}$$

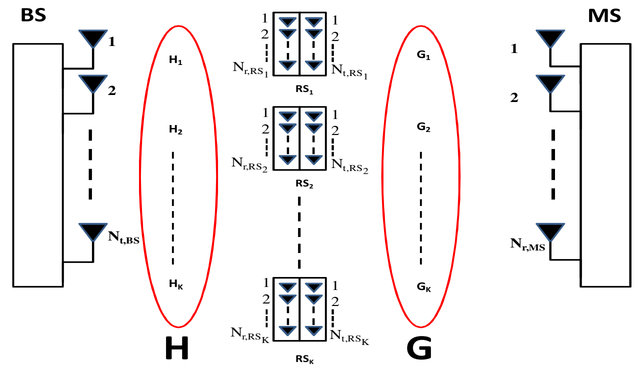

Figure 2: IEEE 802.16j dual-hop relay configuration with multi-antenna parallel relays

where $err(\varphi)$, $err(k)$, $err(l,m)$ and $err(\Omega)$ respectively represent no error, one error $(\tilde{x}_k \neq x_k)$, two errors $(\tilde{x}_l \neq x_l, \tilde{x}_m \neq x_m)$, and all errors in $\bar{\mathbf{x}}_i$. Let $err(\cdot)$ represents an error event of $\bar{\mathbf{x}}_i$, $P(\tilde{x}_i \neq x_i \mid err(\cdot))$ denotes the SEP of x_i conditioned on $err(\cdot)$ and $P(err(\cdot))$ stands for the probability of $err(\cdot)$ [1]. When equal power is allocated among all the transmitted signals, the SEP of each transmitted signal is the same and is given by [1, Eq. (20)].

$$e = \sum_{j=0}^{N_{t,BS}-1} \binom{N_{t,BS}-1}{j} \beta_j e^j (1-e)^{N_{t,BS}-j-1} \tag{4}$$

where $\beta_0 = P(x_i \neq \tilde{x}_i \mid err(\varphi))$ and $\beta_j = P(x_i \neq \tilde{x}_i \mid err(l_1,\cdots,l_j))$, $(l_1,\cdots,l_j)$ is a j-element subset of $\{1,\cdots,i-1,i+1,\cdots,N_{t,BS}\}$. The SEP e can be obtained by solving Eq. (4) for e [1].

B. *Two-hop MIMO Multiplexing System:*

This subsection presents the extension of the accurate SEP analysis for MIMO MLD in [1] to MIMO relaying with MLD at RS and MS. Our objective is to use DF protocol in order to examine the best SEP performance possible using relaying to enhance the data rate. Consider a MIMO relaying system where the BS equipped with $N_{t,BS}$ transmit antennas communicates with an MS equipped with $N_{r,MS}$ receive antennas via RS equipped with $N_{r,RS}$ receive and $N_{t,RS}$ transmit antennas. $\mathbf{H}$ and $\mathbf{G}$ represent the channel matrices of the first and second hop transmission respectively with $(j,i)^{th}$ elements assumed to be an i.i.d complex Gaussian variable with zero mean and unit variance ($\sigma_h^2 = \sigma_g^2 = 1$). As the RS is employing DF protocol, the post detection SNR for the first and second hop transmission conditioned no error $err(\varphi)$, one error $err(k)$, two errors $err(l,m)$ and all errors $err(\Omega)$ in the other symbols can be found in a similar way as described in [1]. These SNR expressions are then used to compute the conditional SEPs. Hence, the average SEPs of the first and second hop transmission can be written respectively as

$$e_{\_hop1} = \sum_{j=0}^{N_{t,BS}-1} \binom{N_{t,BS}-1}{j} \beta_j e_{\_hop1}^j (1-e_{\_hop1})^{N_{t,BS}-j-1} \tag{5}$$

$$e_{\_hop2} = \sum_{j=0}^{N_{t,RS}-1} \binom{N_{t,RS}-1}{j} \beta_j e_{\_hop2}^{j} (1-e_{\_hop2})^{N_{t,RS}-j-1} \quad (6)$$

As the transmitted signal undergoes two states of decoding in cascade, the overall SEP is given by [6]

$$e_{\_tot} = e_{\_hop1} + e_{\_hop2} - e_{\_hop1} e_{\_hop2} \quad (7)$$

Where $e_{\_hopi}$ is the average SEP of the i^{th} ($i = 1,2$) hop. Next we derive the closed-form expressions for $e_{\_hop1}$ and $e_{\_hop2}$ for relay-multiplexing systems. To the best of authors knowledge, these expressions are new contributions.

III. Relay Multiplexing Methods for Multi-Antenna Parallel Relay Stations

This section derives the SEP performance of the proposed relay multiplexing method for IEEE 802.16j system. Consider that the IEEE802.16j system in Fig.1 has the configuration shown in Fig.2, where the BS and the MS, equipped with $N_{t,BS}$ and $N_{r,MS}$ antennas respectively, are communicating via K multi-antenna RSs. The first hop channel between BS and RS is partitioned into K parallel MIMO channels for relay access. The data stream at the BS is then partitioned into K substreams and these substreams are transmitted simultaneously over these K parallel MIMO relay channels. We call this approach relay-multiplexing scheme and it can be employed in addition to the MIMO multiplexing on the multiple antennas. It is assumed that the j^{th} RS, RS_j, has N_{r,RS_j} receiving and N_{t,RS_j} transmitting antennas such that $N_{t,BS} = N_{r,MS} = \sum_{j=1}^{K} N_{r,RS_j} = \sum_{j=1}^{K} N_{t,RS_j}$, and that all K RSs employ DF protocol. $\mathbf{H}_j$ is the channel matrix between the BS and RS_j, and $\mathbf{G}_j$ is the channel matrix between RS_j and the MS. It is also assumed that the BS has perfect knowledge of its forward channels. The BS transmits K independent data streams $\mathbf{z}_1, \mathbf{z}_2, \cdots\cdots, \mathbf{z}_K$ towards the K parallel RSs, where $\mathbf{z}_j = [z_{j,1} \quad z_{j,2} \quad \ldots \quad z_{j,N_{r,RS_j}}]^T$ is the sub-stream destined to RS_j. $\mathbf{z}_j$ is preprocessed before its transmission by multiplying it with ($N_{t,BS} \times N_{r,RS_j}$) preprocessing matrix $\mathbf{W}_{t_j}$

$$\mathbf{x}_j = \mathbf{W}_{t_j} \mathbf{z}_j, \qquad j = 1, 2, \ldots\ldots, K \quad (8)$$

After preprocessing, the $N_{t,BS} \times 1$ signal vector $\mathbf{x} = [x_1, \cdots, x_{N_{t,BS}}]$ broadcast by the BS to K RSs can be expressed as

$$\mathbf{x} = \sum_{j=1}^{K} \mathbf{x}_j = \sum_{j=1}^{K} \mathbf{W}_{t_j} \mathbf{z}_j \quad (9)$$

The $N_{r,RS_j} \times 1$ vector $\mathbf{y}_j$ received at RS_j is expressed as:

$$\mathbf{y}_j = \mathbf{H}_j \mathbf{x} + \mathbf{n}_j = \mathbf{H}_j \mathbf{W}_{t_j} \mathbf{z}_j + \sum_{i=1, i\neq j}^{K} \mathbf{H}_j \mathbf{W}_{t_i} \mathbf{z}_i + \mathbf{n}_j \quad (10)$$

$$= \mathbf{H}_j \mathbf{W}_{t_j} \mathbf{z}_j + \mathbf{H}_j \overline{\mathbf{W}}_{t_j} \overline{\mathbf{z}}_j + \mathbf{n}_j \quad (11)$$

where $\mathbf{n}_j$ is ($N_{r,RS_j} \times 1$)-element AWGN vector with zero mean and unit variance. $\mathbf{H}_j$ is channel matrix from the BS to RS_j; whereas $\mathbf{H}_j \overline{\mathbf{W}}_{t_j} \overline{\mathbf{z}}_j$ is the interference term at RS_j. And $\overline{\mathbf{W}}_{t_j}$ and $\overline{\mathbf{z}}_j$ are respectively the pre-coding matrix and data streams of all other RSs excluding RS_j, and is given by

$$\overline{\mathbf{W}}_{t_j} = [\mathbf{W}_{t_1} \quad \cdots \quad \mathbf{W}_{t_{j-1}} \quad \mathbf{W}_{t_{j+1}} \quad \cdots \quad \mathbf{W}_{t_K}] \quad (12)$$

$$\overline{\mathbf{z}}_j = [\mathbf{z}_1^T \quad \cdots \quad \mathbf{z}_{j-1}^T \quad \mathbf{z}_{j+1}^T \quad \cdots \quad \mathbf{z}_K^T]^T \quad (13)$$

To eliminate the multiple relay-access interference in Eq. (11), we impose the constraint that $\mathbf{H}_j \mathbf{W}_{t_i} = 0$ for $i \neq j$ [8]. If we define $\overline{\mathbf{H}}_j$ as

$$\overline{\mathbf{H}}_j = [\mathbf{H}_1^T \quad \cdots \quad \mathbf{H}_{j-1}^T \quad \mathbf{H}_{j+1}^T \quad \cdots \quad \mathbf{H}_K^T]^T \quad (14)$$

then the zero-interference constraint forces $\mathbf{W}_{t_j}$ ($j = 1, \ldots, K$) to lie in the null space of $\overline{\mathbf{H}}_j$. The singular value decomposition (SVD) of $\overline{\mathbf{H}}_j$ is given as

$$\overline{\mathbf{H}}_j = \overline{\mathbf{U}}_j [\overline{\mathbf{\Lambda}}_j^{1/2} \quad 0] \overline{\mathbf{V}}_j^H = \overline{\mathbf{U}}_j [\overline{\mathbf{\Lambda}}_j^{1/2} \quad 0] \begin{bmatrix} \overline{\mathbf{V}}_{js}^H \\ \overline{\mathbf{V}}_{jn}^H \end{bmatrix} \quad (15)$$

where $\overline{\mathbf{V}}_{js}$ and $\overline{\mathbf{V}}_{jn}$ are the eigenvectors corresponding to the nonzero and zero eigenvalues of $\overline{\mathbf{H}}_j \overline{\mathbf{H}}_j^H$ respectively, and $\overline{\mathbf{U}}_j$ are the eigenvectors of $\overline{\mathbf{H}}_j^H \overline{\mathbf{H}}_j$. Thus, $\overline{\mathbf{V}}_{jn}$ forms an orthogonal basis for the null space of $\overline{\mathbf{H}}_j$ and its columns are the candidate for the precoding matrix $\mathbf{W}_{t_j}$ for the data stream destined to RS_j. Hence, setting $\mathbf{W}_{t_j} = \overline{\mathbf{V}}_{jn}$, the signal received at the RS_j can be expressed as

$$\mathbf{y}_j = \mathbf{H}_j \overline{\mathbf{V}}_{jn} \mathbf{z}_j + \mathbf{n}_j = \mathbf{F}_j \mathbf{z}_j + \mathbf{n}_j \quad (16)$$

where $\mathbf{F}_j = \mathbf{H}_j \overline{\mathbf{V}}_{jn}$ is the effective channel matrix between the BS and RS_j.

A. SEP Analysis for First Hop Detection at RS

The ML detection at RS_j can be written as

$$\tilde{\mathbf{z}}_j = \arg\min_{\mathbf{z}_j \in \chi} \sum_{k=1}^{N_{r,RS_j}} | y_{j,k} - \sum_{l=1}^{(N_{t,BS}/K)} f_{j,k,l} z_{j,l} |^2 \quad (17)$$

As $N_{t,BS}$ is the total number of symbols transmitted from the BS and as there are K RSs, $N_{t,BS}/K$ is the number of symbols destined to RS_j. $f_{j,k,l}$ is the effective channel gain at the k^{th} antenna of RS_j for the l^{th} symbol and it is the (k,l) element of $\mathbf{F}_j$. $\tilde{\mathbf{z}}_j$ represents the decision vector for $\mathbf{z}_j = [z_{j,1} \quad z_{j,2} \quad \ldots \quad z_{j,N_{r,RSj}}]^T$ and $\tilde{z}_{j,l}$ represents the decision symbol for $z_{j,l}$; where $\mathbf{z}_j$ is the sub-stream sent to

RS_j and $z_{j,l}$ is the l^{th} symbol of that sub-stream, detected at the l^{th} antenna element of RS_j. Let $\bar{\mathbf{z}}_{j,l} = \{z_{j,1}, \cdots, z_{j,l-1}, z_{j,l+1}, \cdots, z_{j,N_{r,RSj}}\}$ be the other received signals at the other antenna element of RS_j. Then the SEP of $z_{j,l}$ can be written in terms of the SEPs conditioned on a set of error events corresponding to $\bar{\mathbf{z}}_{j,l}$ and the probabilities of these error events, given by [1]

$$
\begin{aligned}
P(\tilde{z}_{j,l} \neq z_{j,l}) &= P(\tilde{z}_{j,l} \neq z_{j,l} \mid err(\varphi))P(err(\varphi)) + \\
&\sum_{\substack{m=1,\\ m\neq l}}^{N_{t,BS}/K} P(\tilde{z}_{j,l} \neq z_{j,l} \mid err(m))P(err(m)) + \\
&\sum_{\substack{n=1,\\ n\neq l}}^{N_{t,BS}/K} \sum_{\substack{o=n+1,\\ o\neq l}}^{N_{t,BS}/K} P(\tilde{z}_{j,l} \neq z_{j,l} \mid err(n,o))P(err(n,o)) + \\
&\cdots + P(\tilde{z}_{j,l} \neq z_{j,l} \mid err(\Omega))P(\Omega)
\end{aligned}
\tag{18}
$$

where $err(\varphi)$, $err(m)$, $err(n,o)$ and $err(\Omega)$ respectively represent no error, one error $(\tilde{z}_m \neq z_m)$, two errors $(\tilde{z}_n \neq z_n, \tilde{z}_o \neq z_o)$, and all errors $(\tilde{z}_{j,1} \neq z_{j,1}, \cdots, \tilde{z}_{j,l-1} \neq z_{j,l-1}, \tilde{z}_{j,l+1} \neq z_{j,l+1}, \cdots, \tilde{z}_{j,N_{r,RSj}} \neq z_{j,N_{r,RSj}})$ in $\bar{\mathbf{z}}_{j,l}$. Under the event $err(\phi)$, no error occurs to other transmitted symbols of the j^{th} sub-stream, the detection of $z_{j,l}$ becomes

$$
\tilde{z}_{j,l} = \arg\min_{z_{j,l}} \sum_{k=1}^{N_{r,RS_j}} | y_{j,k} - f_{j,k,l} z_{j,l} |^2 \tag{19}
$$

The post detection SNR is given as

$$
\begin{aligned}
\gamma_{j,l,err(\phi)} &= \sum_{k=1}^{N_{r,RS_j}} \gamma_{j,k,l,err(\varphi)} \\
&= \frac{\sum_{k=1}^{N_{r,RS_j}} | f_{j,k,l} |^2 | z_{j,l} |^2}{\sigma_n^2} = \frac{\omega_{j,l} | z_{j,l} |^2}{\sigma_n^2}
\end{aligned}
\tag{20}
$$

where $\omega_{j,l} = \sum_{k=1}^{N_{r,RS_j}} | f_{j,k,l} |^2$ and σ_n^2 is the noise variance at RS_j. For 4QAM modulated signals, the SEP conditioned on $z_{j,l}$, $\omega_{j,l}$ and $err(\phi)$ can be written using [1, Eq. (9)], and then averaging the result over the statistics of $z_{j,l}$ and $\omega_{j,l}$. The average SEP conditioned on no error occurs in other transmitted symbols is thus derived as

$$
\begin{aligned}
&P(\tilde{z}_{j,l} \neq z_{j,l} \mid err(\varphi)) \\
&= \frac{1}{\pi} \sum_{z_{j,l} \in \chi} p(z_{j,l}) \left(\frac{3\pi}{4} + \sum_{i=1}^{N_{r,RS_j}} (-1)^i \sum_{r=0}^{i-1} \frac{a_{j,l,err(\varphi)}^{i-r-1/2}}{(1 + a_{j,l,err(\varphi)})^{i-1/2}} \right. \\
&\left. . \binom{N_{r,RS_j}}{i} . \binom{i-1}{r} . I\left(0, -\sqrt{\frac{1 + a_{j,l,err(\varphi)}}{a_{j,l,err(\varphi)}}}, (r+1) \right) \right)
\end{aligned}
\tag{21}
$$

where $a_{j,l,err(\varphi)} = {|z_{j,l}|^2}/{2\sigma_n^2}$. Similarly the SEP conditioned on one error $P(\tilde{z}_{j,l} \neq z_{j,l} \mid err(m))$, two errors $P(\tilde{z}_{j,l} \neq z_{j,l} \mid err(n,o))$ and all errors $P(\tilde{z}_{j,l} \neq z_{j,l} \mid err(\Omega))$ in other transmitted symbols can be derived similar to Eq. (21), but the variable $a_{j,l,err(\phi)}$ is replaced with $a_{j,l,err(m)}$, $a_{j,l,err(n,o)}$, $\cdots$, $a_{j,l,err(\Omega)}$ respectively; where

$$
\begin{aligned}
a_{j,l,err(m)} &= \frac{| z_{j,l} |^2}{2\sigma_{v,m}^2} = \frac{| z_{j,l} |^2}{2(| \Delta z_m |^2 + \sigma_n^2)}, \\
a_{j,l,err(n,o)} &= \frac{| z_{j,l} |^2}{2\sigma_{v,n,o}^2} = \frac{| z_{j,l} |^2}{2(| \Delta z_n |^2 + \Delta z_o |^2 + \sigma_n^2)}, \\
&\vdots \\
a_{j,l,err(\Omega)} &= \frac{| z_{j,l} |^2}{2\sigma_\Omega^2} \\
&= \frac{| z_{j,l} |^2}{2(| \Delta z_{j,1} |^2 + \cdots + | \Delta z_{j,l-1} |^2 + | \Delta z_{j,l+1} |^2 + \cdots + | \Delta z_{j,N_{r,RSj}} |^2 + \sigma_n^2)}
\end{aligned}
\tag{22}
$$

where $\Delta z_{j,k} (k = 1,2,\cdots, N_{r,RS_j}, k \neq l)$ can be approximated by $| \Delta z_{j,k} |^2 \approx \min(d_{j,k}^2) = \alpha_{j,k} E\{| z_{j,k} |^2\}$, with $\min(d_{j,k}^2)$ representing the minimum squared Euclidean distance (SED) between $z_{j,k}$ and its constellation neighbors and $\alpha_{j,k}$ stands for the ratio of $\min(d_{j,k}^2)$ to the average transmit power of $z_{j,k}$. It is easily noticeable that $\min(d_{j,k}^2)$ and $\alpha_{j,k}$ vary with the modulation scheme. For 4QAM modulation, $\min(d_{j,k}^2) = 2E\{| z_{j,k} |^2\}$ [1]. The probabilities $P(err(\phi))$, $P(err(m)), P(err(n,o)), \cdots, P(err(\Omega))$ in Eq. (18) can be expressed similar to [1, Eq. (19)] for our case here as

$$
\begin{aligned}
P(err(\phi)) &= \prod_{k=1,k\neq l}^{N_{t,BS}/K} (1 - P(\tilde{z}_{j,l} \neq z_{j,l})) \\
P(err(m)) &= P(\tilde{z}_{j,m} \neq z_{j,m}). \prod_{k=1,k\neq l,m}^{N_{t,BS}/K} (1 - P(\tilde{z}_{j,l} \neq z_{j,l})) \\
P(err(n,o)) &= .P(\tilde{z}_{j,n} \neq z_{j,n}) P(\tilde{z}_{j,o} \neq z_{j,o}) \prod_{k=1,k\neq l,m,n}^{N_{t,BS}/K} (1 - P(\tilde{z}_{j,l} \neq z_{j,l})) \\
&\vdots \\
P(err(\Omega)) &= \prod_{k=1,k\neq l}^{N_{t,BS}/K} (P(\tilde{z}_{j,l} \neq z_{j,l}))
\end{aligned}
\tag{23}
$$

When equal power is allocated among all the transmitted signals, the SEP of each transmitted symbol is the same under i.i.d fading case. Thus the SEP at RS_j can be expressed, after substituting Eq. (23), Eq. (21) and modified versions of Eq. (21) according to Eq. (22), into Eq. (18) and solving, as

$$e_j = \sum_{i=0}^{\frac{N_{t,BS}}{K}-1} \binom{\frac{N_{t,BS}}{K}-1}{i} \beta_i e_j^i \left(1-e_j\right)^{\frac{N_{t,BS}}{K}-i-1} \tag{24}$$

where e_j denotes the SEP at RS_j. The SEP e_j at RS_j can be obtained by solving Eq. (24) for e_j. The average SEP for the first hop transmission in the relay-multiplexing system can be written as

$$e_{\_hop1} = \sum_{j=1}^{K} P(\mathbf{z}_j) e_j \tag{25}$$

where $P(\mathbf{z}_j)$ is the probability that the sub-stream $\mathbf{z}_j$ is transmitted and e_j is the SEP of the sub-stream $\mathbf{z}_j$. Assuming all sub-streams are of equal length, and they experience same fading, the average SEP of the first hop transmission in relay-multiplexing can be written as

$$e_{\_hop1} = \frac{1}{K} \sum_{j=1}^{K} e_j \tag{26}$$

B. SEP Analysis for Second Hop Detection at MS

The K data streams received at the K RSs are decoded and then simultaneously forwarded to the MS during the second hop transmission. Hence, the transmission from K parallel RSs to the MS can be treated as $\sum_{j=1}^{K} N_{t,RS_j} \times N_{r,MS}$ conventional MIMO multiplexing system. The SEP for the second hop transmission (i.e. from K RSs to the MS) can thus be calculated in a similar manner as described earlier in section II, but here for an $\sum_{j=1}^{K} N_{t,RS_j} \times N_{r,MS}$ MIMO multiplexing system. Let $e_{\_hop2}$ denotes the average SEP of the second hop transmission, then we can derive an expression for $e_{\_hop_2}$ as

$$e_{\_hop2} = \sum_{i=0}^{\sum_{j=1}^{K} N_{t,RS_j}-1} \binom{\sum_{j=1}^{K} N_{t,RS_j}-1}{i} \beta_i e_{\_hop2}^i \left(1-e_{\_hop2}\right)^{\sum_{j=1}^{K} N_{t,RS_j}-i-1} \tag{27}$$

The SEP $e_{\_hop2}$ for the second hop transmission can then be obtained by solving Eq. (27) for $e_{\_hop2}$. Finally, the overall SEP $e_{\_tot}$ of MIMO-relay multiplexing system shown in Fig. 2 can be expressed as in Eq. (7) above.

IV. SIMULATION RESULTS

This section presents our simulation results and their comparison with the analytical results derived in the previous sections. It should be noted that 4QAM modulation scheme is used, RSs assumed in the simulation employ DF protocol and flat Rayleigh fading is also assumed in all the configurations considered. Fig. 3 and 4 present the SEP results for MIMO multiplexing and two-hop MIMO multiplexing systems respectively. It is observed from the figure that our analytical results agree very closely with the simulations for both cases confirming the validity of our analysis. Notice also that the results in Fig. 3 agree closely with the equivalent result in [1] for the case of 2x2 MIMO systems.

In Fig. 5 and 6, we present summary of our SEP results for three different cases of MIMO-relay multiplexing scheme. For Case 1, BS and MS are equipped with four antennas each and two parallel RSs with two antennas each are assumed in between BS and MS. For Case 2 and 3, BS and MS are equipped with six antennas each, where three parallel RSs with two antennas each are assumed for Case 2, and two parallel RSs with three antennas each are assumed for Case 3. Fig. 5 presents the hop-wise SEP for these three cases, while Fig. 6 presents the overall SEP at the MS for the three cases. It can be observed from the Fig. 5 that the SEP for MIMO-relay multiplexing system in Case 1 and 2 for the first hop are the same due to the fact that for these two cases, the first hop channel is partitioned into two and three parallel 2x2 MIMO channels respectively, while for Case 3, the first hop channel is partitioned into two 3x3 parallel MIMO channels. As the overall SEP for the first hop is the average of these parallel MIMO channels, we have better SEP results for Case 3 compared to Cases 1 and 2. Fig. 5 also presents the SEP curves for the second hop transmissions for Cases 1, 2 and 3 where channels from RSs to MS are treated as equivalent to 4x4, 6x6 and 6x6 conventional MIMO multiplexing systems respectively. As expected, Cases 2 and 3 have same SEP, and are better than Case 1, in the second hop transmission.

In Fig. 6, we present the overall SEP at MS for these three cases. Comparing the results in this figure with the hop-wise analysis in Fig. 5, it can be noticed that the SEP in the first hop dominates the overall error rate of the MIMO-relay multiplexing system for all three cases considered. As for Cases 1 and 2, the SEP in the first hop is the average of parallel 2x2 MIMO channels, so they exhibits higher SEP compared to the Case 3, where the SEP in the first hop is the average of parallel 3x3 MIMO channels. Although, for Case 1 and 2 we have different equivalent MIMO configurations in the second hop, they have very close SEP curves due to the fact that they have same SEP in the first hop and it dominates the overall SEP. Also, Case 3 exhibits better SEP as its SEP in the first hop is better than those of Cases 1 and 2.

In general, we observe from our results in Fig. 5 and 6 that MIMO-relay multiplexing approach is practicable, when the number of antennas at the BS and MS are equal or greater than the total number of antennas on all the RSs. In such scenarios, the parallel relaying of independent data streams via multiple parallel RSs offers acceptable SEP performance while boosting the capacity of the system linearly with the number of parallel RSs as discussed in [7]. We also observe that for $K \geq 2$, increasing the number of antennas at RSs works better for SEP enhancement in relay-multiplexing systems than increasing the number of RSs, for the same parallel number of antennas at the BS and MS, as can be observed from the relative performance of the system for cases 2 and 3 in our results.

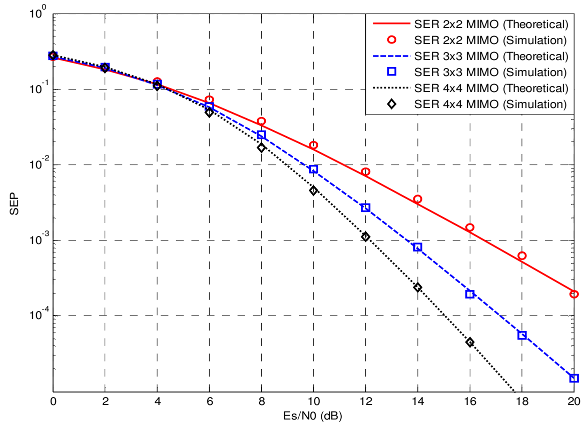

Figure 3: SEP for 2x2, 3x3 and 4x4 MIMO multiplexing

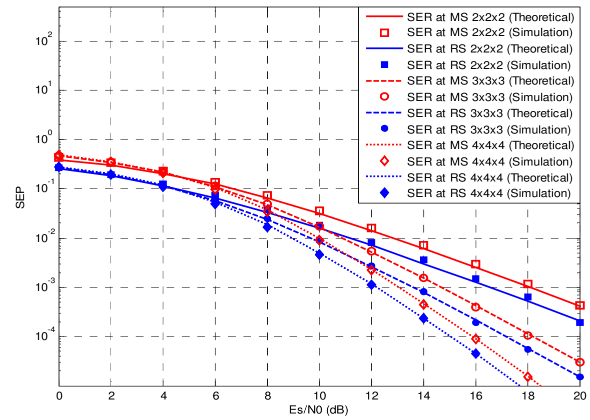

Figure 4: SEP for two-hop MIMO multiplexing

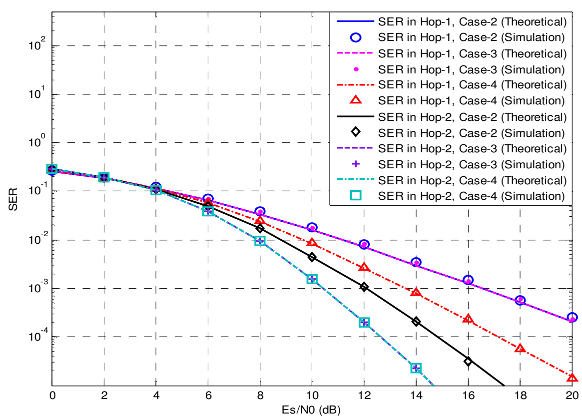

Figure 5: Hop-wise SEP for MIMO relay multiplexing

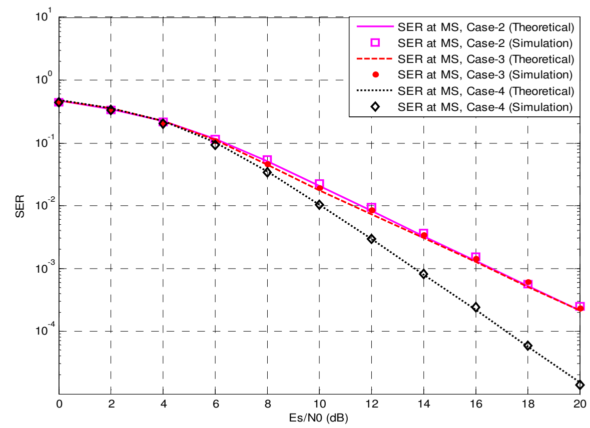

Figure 6: Overall SEP for MIMO relay multiplexing

V. CONCLUSIONS

This paper provides the SEP analysis of MIMO relay-multiplexing technique for IEEE 802.16j systems. We derive closed-form expressions that are verified by computer simulations. Based on our results, it is concluded that relay-multiplexing approach, when applied with proper relay-access transmission techniques at the BS, can provide reliable parallel and independent data transmission paths via the different RSs. It is also concluded that for $K \geq 2$, the SEP performance of relay-multiplexing scheme can be better enhanced by increasing the number of antennas at the RSs, than by increasing the number of RSs, for the same overall number of antennas. It is hoped that these findings provide useful guides for relay-multiplexing deployments in IEEE 802.16j systems.

ACKNOWLEDGMENT

This work is supported by a grant (no. 09-ELE928-02) from the National Plan for Science and Technology (NPST), King Saud University, Saudi Arabia.

REFERENCES

[1] W. Peng, S. Ma, T. Ng, and J. wang, "A Novel Analytical Method for Maximum Likelihood Detection in MIMO Multiplexing Systems," IEEE Trans. on Commun., vol. 57, no. 8, pp. 2264-2268, Aug. 2009.

[2] M.O. Hasna and M.-S. Alouini, "End-to-End Performance of Transmission Systems With Relays Over Rayleigh-Fading Channels," IEEE Trans. on Wireless Commun., vol. 2, no. 6, Nov. 2003.

[3] P.A. Anghel and M. Kaveh, "Exact symbol error probability of a cooperative network in a Rayleigh-fading environment," IEEE Trans. Wireless Commun., vol. 3, no. 5, pp. 1416-1421, Sept. 2004.

[4] S. Ikki and M.H. Ahmad, "Performance analysis of cooperative diversity wireless networks over nakagami-m fading channel," IEEE Commun. Letters, vol. 11, no. 4, pp. 334-336, July 2007.

[5] S.S. Ikki and M.H. Ahmed, "Multi-branch decode-and-forward cooperative diversity networks performance analysis over Nakagami-m fading channels," IET Communications, vol.5, no.6, pp.872-878, April 15 2011.

[6] A. Gharanjik, K. Mohamed-pour, S.M.H. Andargoli, and M. Hoseinzade, "End-to-end performance of dual hop transmission with fixed relay," in Wireless Telecommun. Symp. (WTS), pp. 1-6, 21-23 April 2010.

[7] A.I. Sulyman, A. Akhlaq, H. Hassanein, A. Alsanie, and S. Alshebeili, "Capacity enhancements in IEEE 802.16J systems using MIMO-relay multiplexing," in GCC Conference and Exhibition (GCC), pp. 13-16, 19-22 Feb. 2011.

[8] Lai-U Choi and R.D. Murch, "A transmit preprocessing technique for multiuser MIMO systems using a decomposition approach," IEEE Trans. on Wireless Commun., vol. 3, no. 1, pp. 20-24, Jan. 2004.