

Dynamic Small Cell Placement Strategies for LTE Heterogeneous Networks

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Abstract—Small cell deployments have proven to be a cost-effective solution to meet the ever growing capacity and coverage requirements of mobile networks. While small cells are commonly deployed indoors, more recently outdoor roll-outs have garnered industry interest to complement existing macrocell infrastructure. However, the problem of where and when to deploy these small cells remains a challenge. In this paper, we investigate the small base station (SBS) placement problem in high demand outdoor environments. First, we propose a dynamic placement strategy (DPS) that optimizes SBS deployment for two different network objectives: 1) minimizing data delivery cost, and 2) minimizing macrocell utilization. We formulate each problem as a mixed integer linear program (MILP) that determines the optimal set of deployment locations among the candidate hot-spots to meet each network objective. Then we develop two greedy algorithms, one for each objective, that achieve close to optimal MILP performance. Our simulation results demonstrate that significant delivery cost and MBS utilization reductions are possible by incorporating the proposed deployment strategies.

Index Terms—Small base station deployment; heterogeneous network design; traffic offloading; data delivery cost.

I. INTRODUCTION

In the last few years, global mobile data traffic has experienced at least a ten-fold growth according to Cisco's Global Visual Networking Index (VNI) [1]. This growth is driven by the proliferation of data-intensive applications such as high definition video streaming, social networking, and online gaming. In addition to that, the number of smartphones and Internet connected devices is growing exponentially and currently exceeds the world's population [1]. Therefore, cellular operators have been searching extensively for solutions to increase capacity and improve coverage to satisfy mobile users, as well as to cope with this explosion in data traffic. As improvements in radio link are approaching theoretical limits, most cellular operators have established that the next performance leap will stem from changing the network topology [2]. Using a mixture of macro base stations (MBSs) overlaid with small cells is referred to as a Heterogeneous Network (HetNet). HetNets are now considered a core part of the 3rd Generation Partnership Project (3GPP) Long Term Evolution (LTE) and LTE-Advanced [2] and enable a significant increase in spectrum reuse per area [3].

A small cell is a cellular coverage area that is served by low-power small base station (SBS) [4]. An SBS is a fully featured mini base station that is typically intended for indoor deployment and backhauled to the operator's core network

(CN) via an Internet connection (such as DSL, cable, etc.) [4], [5]. Small cell deployments include femtocells, picocells and metrocells. Recently, however, several operators are starting outdoor deployments [5], and recent research efforts have proposed to deploy small cells in public transportation vehicles including buses and streetcars [6]. SBSs can be used to offer enhanced capacity at high demand areas (hot-spots) and thereby offload traffic from macrocells [7] [8]. Due to their potential benefits, small cell deployments have generated significant interest in the mobile industry and academia/research bodies. In fact, the total number of already deployed small cells has exceeded the total number of macrocells [5].

Adopting small cells generates two significant challenges. First, an exhaustive deployment of SBS in all regions of interest (especially outdoors) is an overkill since not all regions necessitate an SBS deployment to meet its demands. Second, given a restrictive deployment strategy (i.e. with a cap on the total number of SBSs to deploy), we are faced with the challenge of where to deploy them to maximize the operator deployment objectives. One objective for example may be to minimize the total cost of service delivery, whereas another may be to minimize the resources/power consumed at the MBSs. Therefore, effective SBS deployment strategies are needed in order to realize the potential benefits of HetNets.

In this paper, we study the problem of optimizing SBS placement in high-traffic urban environments to complement macrocells serving outdoor users. We optimize SBS deployment for two different objectives 1) minimizing delivery cost, and 2) minimizing macrocell utilization. In our solutions we incorporate information of the requested and achievable rates at each candidate site, in addition to other deployment constraints. Our main contributions in this paper are:

- We propose a Dynamic Placement Strategy (DPS) for SBS deployment that exploits knowledge of traffic demand and achievable throughput at the candidate sites (hot-spots). Two DPS problems are formulated as mixed integer linear programs (MILP) for the different deployment objectives. These MILPs provide benchmark solutions for the DPS problem.
- We propose two greedy algorithms for the formulated DPS problems. Extensive simulations indicate that the algorithms achieve close to optimal results compared to the DPS MILP-based benchmark solutions.

As a result of the proposed dynamic placement strategies, the overall mobile user experience is enhanced while minimizing the additional associated costs.

The remainder of this paper is outlined as follows. Section II overviews the related work. In Section III, we describe our system model and elaborate on the link and traffic models. The proposed DPS MILP formulations and the corresponding greedy algorithms are developed in Section IV. The performance evaluation is elaborated upon in Section V, and we conclude this work in Section VI.

II. RELATED WORK

Small cell deployments are gaining high interest in industry and academia due to their diverse benefits. The authors of [9] studied the performance of co-channel LTE-Advanced HetNets and their results show that there is a significant increase in the network capacity when pico-cells are deployed. Similarly, the work in [10] presents a simple study of an LTE scenario with one MBS and one pico BS to demonstrate that picocells are able to increase network capacity and reduce power consumption.

Although there are many research efforts addressing the benefits of small cell deployments, works targeting SBS *placement optimization* for LTE outdoor scenarios remain limited. Among the works addressing *indoor* optimal deployment of small cells are [11] and [12]. In [11], I. Ahmed *et al.* propose a genetic placement algorithm for airport environments to serve traffic demand and minimize outage and power consumption. On the other hand, the work presented in [12] studies the femtocell placement problem in commercial buildings. The objective is to minimize the power consumption of user equipment (UE) while covering all areas in a building.

The work in [13] is closest to our work. The authors propose a sampling based optimization method for 3rd Generation (3G) small cell deployments. SBSs deployment is optimized with the objective of maximizing UEs throughput. As opposed to [13] which focuses on UE throughput, we discuss the problem of optimizing deployment to achieve network-wide objectives. We formulate placement strategies that 1) minimize the service delivery cost (by maximizing the offloaded traffic),

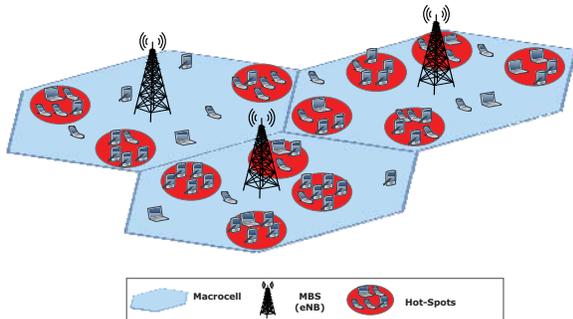


Fig. 1. An instance of the considered network.

TABLE I
SUMMARY OF IMPORTANT SYMBOLS

Symbol	Description
i	Candidate site index $i = \{1, 2, \dots, C\}$
j	eNB index $j = \{1, 2, \dots, M\}$
\mathcal{C}	Set of candidate sites
\mathcal{M}	Set of eNBs in the network
N	Total number of SBSs that can be deployed
x_i	The fraction of BS air-time allocated to candidate site i
s_i	Indicator variable representing SBS installation at candidate site i
D_i	Demand at candidate site i [Mbps]
R_i	Achievable throughput at candidate site i [Mbps]
\mathcal{U}_j	Set of indices of candidate sites in the coverage area of macrocell j

and 2) minimize MBS utilization, thereby providing more resources for additional services in the macrocell.

III. SYSTEM OVERVIEW

In this section, we present the notations used in this paper, as well as our network and traffic models.

A. Notations

We use the following notational conventions: \mathcal{X} denotes a set and its cardinality $|\mathcal{X}|$ is denoted by X . \vec{x} is used to denote vectors, e.g., $\vec{x} = (x_a : a \in \mathcal{A})$. The frequently used symbols are summarized in Table 1.

B. Network Model

An instance of our network model is represented in Fig. 1. The indicated hot-spots are of concern to mobile operators due to the constant high demand in these geographical regions. In this paper, we optimize the SBSs placement among these candidate sites based on the network objective. We consider downlink (DL) transmission in the LTE HetNet that consists of a set of MBSs, or evolved Node B (eNBs), denoted by the set $\mathcal{M} = \{1, 2, \dots, M\}$. The candidate sites where SBSs can be deployed are denoted by the set $\mathcal{C} = \{1, 2, \dots, C\}$. An arbitrary eNB is denoted by $j \in \mathcal{M}$ and a candidate site by $i \in \mathcal{C}$. We define the set \mathcal{U}_j which contains the indices of all the candidate sites that are in the coverage area of eNB j .

We assume that eNBs and small cells operate on different dedicated frequency carriers [14] [15] in order to neglect the frequency interference between the two tiers. For the sake of simplicity we also assume that there is enough distance between each candidate site and the others to eliminate frequency interference between small cells. Finally, at each candidate site a backhaul and power source can be set-up to facilitate the deployment.

C. Link and Traffic Models

We denote the requested peak traffic demand at each candidate site i as D_i [Mbps], where $\vec{D} = (D_i : i \in \mathcal{C})$. It is assumed that this demand is known based on service provider network monitoring tools. To determine the average path loss

at each candidate site, we consider the following path loss model:

$$PL_i(d_i) = 128.1 + 37.6 \log_{10} d_i \quad (1)$$

where d_i is the distance in km between the center of candidate site i and its associate eNB (i.e. the closest macrocell). Hence, the achievable throughput at each candidate site can be approximated using Shannon's capacity equation with signal-to-noise ratio (SNR) clipping at $20dB$ for practical modulation orders as follows:

$$R_i = B \log_2(1 + P_i^{rx}/N_0B) \quad (2)$$

where R_i is the data rate at candidate site i , B is the eNB bandwidth, P_i^{rx} is the received power at site i (computed using the PL model of (1)), and N_0 is the background noise power spectral density. Therefore, the vector of achievable rates at each candidate site is denoted by $\vec{R} = (R_i : i \in \mathcal{C})$.

Each MBS j can use its air-time to serve the macrocell traffic and the traffic demanded at the candidate sites (hot-spots). The fraction of air-time during 1 second that is required to serve the macrocell users (not in the hot-spots) is denoted by B_j ; which is assumed to be known based on network monitoring tools. This will provide a remaining air-time fraction of $1 - B_j$ to serve the different hot-spots.

IV. SMALL CELL DYNAMIC PLACEMENT STRATEGIES

The main objective of this work is to determine the optimal locations to deploy a limited number of SBSs among a set of candidate hot-spots in a network of macrocells. We have two network goals: 1) to minimize the service delivery cost and 2) to minimize the macrocell resources consumed. Toward this end, we propose dynamic placement strategies which are first formulated as two Mixed Integer Linear Programs (MILPs) to provide benchmark solutions. Then, we develop two greedy algorithms for each network objective that achieve close to optimal performance.

A. Decision Variables

We introduce a decision variable s_i to indicate if a SBS will be installed at candidate site i . Therefore s_i is defined as follows:

$$s_i = \begin{cases} 1, & \text{if a SBS is deployed at candidate site } i \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

We also define an air-time decision variable x_i which represents the fraction of BS air-time (during 1 second) that is allocated to candidate site i . Since the achievable throughput at site i is R_i [Mbps], the transmitted data during 1 second will be $x_i R_i$.

B. DPS: Optimal Problem Formulations

1) **DPS-Minimizing Delivery Cost (DPS-MinCost)**: The objective of DPS-MinCost formulation is to minimize the data delivery cost of network traffic. Using the optimization variables defined in Section IV-A, the total data delivered per second through the MBSs can be expressed as

$\sum_{j=1}^M \sum_{\forall i \in \mathcal{U}_j} x_i R_i$; whereas the total data delivered per second through the SBSs is $\sum_{i=1}^C s_i D_i$. Note that it is assumed that the SBS backhaul is larger than the demanded traffic at each site, i.e. larger than $\max(D_i)$. The cost of delivering the data is assumed to be proportional to the amount of data transmitted, with the delivery cost through SBSs expressed as a fraction of the cost through MBSs as presented in [16] and [17]. We denote this factor by γ , where common values for γ are 3–5. With these definitions, the DPS-MinCost problem can be formulated as:

$$\begin{aligned} & \underset{x, s}{\text{minimize}} && \left(\sum_{j=1}^M \sum_{\forall i \in \mathcal{U}_j} x_i R_i + \sum_{i=1}^C s_i D_i / \gamma \right) && (4) \\ & \text{subject to:} && \text{C1: } \sum_{i=1}^C s_i \leq N, \\ & && \text{C2: } \sum_{\forall i \in \mathcal{U}_j} x_i \leq 1 - B_j, \quad \forall j \in M, \\ & && \text{C3: } x_i R_i + s_i D_i \geq D_i, \quad \forall i \in \mathcal{C}, \\ & && \text{C4: } 0 \leq x_i \leq 1, \quad s_i \in \{0, 1\}. \end{aligned}$$

The objective function minimizes the delivery cost by deploying SBSs in hot-spots with high demands. Note that this is also equivalent to maximizing the amount of offloaded traffic, i.e. traffic delivered through the SBSs. Constraint C1 ensures that the total number of deployed SBSs is less or equal to the maximum number of SBSs that the operator can deploy, which is denoted by N . Constraint C2 limits the allocated air-time to all SBS served by MBS j to $1 - B_j$, where B_j is the air-time used for the MBS traffic. The purpose of Constraint C3 is to ensure that each candidate site receives its requested demand. As indicated in the constraint, this can come from either the MBS or the deployed SBS. Finally, C4 defines the domain of the decision variables. By solving (4) the optimal subset of candidate sites will be selected for deployment, and the remaining hot-spots will be served by the MBSs.

2) **DPS-Minimizing MBS Utilization (DPS-MinUtil)**: The formulation in (4) minimizes the data delivery cost, but does not necessarily minimize the load at the macro-cell. This may be another objective, where lower macrocell load corresponds to less downlink power consumption, or more resources for other services. In order minimize MBS resource utilization, the sites that require significant MBS air-time will be selected for deployment. The emphasis here is the ratio between the demand and the achievable rate for each site, i.e. D_i/R_i . Therefore, a site with a moderate demand maybe selected for deployment if it has a low R_i (indicating that it is located at the cell edge). The DPS-MinUtil problem can therefore be formulated as the following MILP:

$$\begin{aligned} & \underset{x}{\text{minimize}} && \sum_{i=1}^C x_i && (5) \\ & \text{subject to:} && \text{C1 to C4.} \end{aligned}$$

Here, the objective is to minimize the sum air-time fractions allocated to serve the SBSs in the network of M MBSs, and similar resource and service constraints hold as in (4). The solution to (5) will determine the optimal subset of candidate sites that minimize the total load of the MBSs. The preceding MILPs provide a solution benchmark but require an optimization solver to generate the results. We therefore present the following corresponding greedy algorithms that achieve close to optimal performance.

C. DPS Greedy Algorithms

1) **Greedy DPS-MinCost Algorithm:** The Greedy DPS-MinCost algorithm is represented in Algorithm 1. The algorithm's objective is to minimize delivery cost of mobile traffic, similar to the DPS-MinCost formulation. The DPS-MinCost algorithm is divided into three stages. The first stage is the pre-selection process (indicated in lines 4-9) where the constraint violating candidate site(s) are included in a pre-selected set. Violating candidate site(s) are the sites that if not considered in the SBS deployment solution \mathcal{S} , will either overload the macrocell resources (C2) or violate the demand satisfaction constraint (C3). The second stage, represented by lines 10-11, continues the selection process of candidate site(s) based on their demands, where the ones with the highest demands are considered first. The third stage, represented by lines 12-15, checks if the resulting candidate site(s) selection \mathcal{S} will not cause an overload to the macrocell resources. If any of the macrocells is overloaded (i.e. air-time consumed ≥ 1), the algorithm will re-select other candidate site(s) within the problematic macrocell to resolve the overloading issue. Similar to the second stage, the re-selection process in the third stage is conducted based on the demand. The rest of the algorithm checks if there is a feasible SBS deployment solution after applying the aforementioned stages. If so, the algorithm returns the viable solution. If not, the highest demand candidate site in the violating macrocell will be added to the pre-selection set and the algorithm re-performs the aforementioned stages.

2) **Greedy DPS-MinUtil Algorithm:** The Greedy DPS-MinUtil algorithm, represented in Algorithm 2, aims to minimize macrocells utilization in the network, i.e. similar to the DPS-MinUtil formulation. As in algorithm 1, the Greedy DPS-MinUtil algorithm has a pre-selection stage (indicated in lines 4-9) where all the violating candidate site(s) are included in a pre-selection set. Unlike algorithm 1, where the remaining candidate site(s) are chosen based on their demands, the Greedy DPS-MinUtil algorithm selects the remaining candidate site(s) based on their fraction of air-time, as indicated in lines 10-11. If the resulting SBS deployment solution does not violate the air-time constraints (as indicated in line 12), the solution is returned as feasible.

V. PERFORMANCE EVALUATION

In this section we present the numerical results that demonstrate the potential of the proposed deployment strategies. We also compare the results of the proposed algorithms to the benchmark MILP results.

Algorithm 1 Greedy DPS-MinCost

1. Input: $\mathcal{M}, \mathcal{C}, \vec{D}, \vec{R}, N, \mathcal{U}_j$
 2. Output: \mathcal{S} {deployment set}
 3. Initial phase: no deployment solution
 4. **for** $j = 1$ to M **do**
 5. check for deployment constraints C2, C3 and C4 in all candidate sites in \mathcal{U}_j
 6. **if** candidate site(s) i violates any constraint **then**
 7. site(s) i are added to the pre-selection set \mathcal{P}
 8. **end if**
 9. **end for**
 10. $\mathcal{S} = \mathcal{P}$
 11. update \mathcal{S} to include $N - |\mathcal{P}|$ additional site that have the highest demand
 12. **for** $j = 1$ to M **do**
 13. check MBS j for the violation of deployment constraints; reallocate SBS(s) on that macrocell j based on its candidate site(s) demands D_i while considering the needed air-time to match the demand
 14. update \mathcal{S} based the reallocation process
 15. **end for**
 16. **if** a deployment violation still persist **then**
 17. add the highest demand candidate site(s) i from violating macrocell j to the set \mathcal{P}
 18. **if** $|\mathcal{P}| \leq N$ **then**
 19. restart Algorithm 1
 20. **end if**
 21. **else**
 22. return \mathcal{S} as valid deployment solution
 23. **end if**
 24. return no feasible solution found
-

Algorithm 2 Greedy DPS-MinUtil

1. Input: $\mathcal{M}, \mathcal{C}, \vec{D}, \vec{R}, N, \mathcal{U}_j$
 2. Output: \mathcal{S} {deployment set}
 3. Initial phase: no deployment solution
 4. **for** $j = 1$ to M **do**
 5. check for deployment constraints C2, C3 and C4 in all candidate sites in \mathcal{U}_j
 6. **if** candidate site(s) i violates any constraint **then**
 7. site(s) i are added to the pre-selection set \mathcal{P}
 8. **end if**
 9. **end for**
 10. $\mathcal{S} = \mathcal{P}$
 11. update \mathcal{S} to include $N - |\mathcal{P}|$ candidate sites with the highest x_i
 12. **if** $\sum_{\forall i \in \mathcal{U}_j} x_i > 1 - B_j \quad \forall j$ **then**
 13. Return \mathcal{S} as the valid deployment solution
 14. **else**
 15. return no feasible solution found
 16. **end if**
-

A. Evaluation Setup

We consider a network with 7 MBSs (or eNBs) and 30 hot-spots (candidate sites). Each eNB has a 0.5 km radius,

TABLE II
SUMMARY OF IMPORTANT PARAMETERS

Parameter	Value
C	30 candidate sites
M	7 MBSs
N	Varied between 12 and 30
Path loss	According to (1)
eNB total transmission power	40 W
eNB inter-site distance	1000 m
Background MBS traffic air-time	Uniformly distributed over [0 0.5]
Candidate site demand D_i	Uniformly distributed over [1 16] [Mbps]

a transmit power of 40W and a transmission bandwidth of 10MHz. The locations of the candidate sites are randomly selected within the macrocells and the traffic demand D_i is randomly generated with a uniform distributed over the interval [1 16] Mbps. A summary of the simulation parameters is provided in Table II. We use MATLAB as a simulation platform and Gurobi Optimization [18] to solve the DPS MILPs. Simulation experiments are repeated 100 times to obtain the average values of following metrics:

- Normalized total cost: the total delivery cost of data in the network, where 1 Mbps costs 1 cost unit through the MBSs and 1/5 units through the SBSs (i.e. $\gamma=5$).
- Macrocell offloaded traffic: the percentage of the total network traffic that is offloaded to the SBSs.
- Macrocell resource utilization: the fraction of the MBS air-time consumed for data delivery.

Note that for a given value of maximum SBS deployments (N), it may not be possible to find a viable deployment solution that satisfies all the site demands \vec{D} , i.e. Constraint C3 in (4). This occurs for instances where N is small and the sites have high data demands. We quantify the percentage of successful SBS deployment solutions for a given N in a deployment success rate metric.

B. Results

Fig. 2 shows the normalized data delivery cost for a varying number of SBS installations N . As expected, the cost decreases with increasing N for all the DPS approaches. This is because delivery through SBSs is lower by the factor γ compared to delivery via MBSs. We also observe that the DPS-MinCost approach achieves a lower cost compared to the DPS-MinUtil approach, but converges as N increases. The reason is that with many SBSs available for deployment, both DPS approaches will have a large overlap in the selected SBSs, and the cost difference will diminish. At $N = 30$, all the SBSs will be selected for installation since $C = 30$. This is also apparent in Fig. 3 which illustrates the macrocell offloaded traffic percentage, where at $N = 30$ all the traffic is offloaded to SBSs. From Fig. 3 we also observe that the lower cost is associated with more traffic being offloaded to the SBSs, which is in agreement with the discussion in Section IV-B1. Figures 2 and 3 also demonstrate how the greedy MinCost and MinUtil algorithms achieve close to optimal results.

In order to investigate the effectiveness of the DPS-MinUtil approach we plot the macrocell utilization fraction in Fig. 4. As indicated, the DPS-MinUtil optimal and algorithm results consume less MBS resources. This is in spite of a higher data delivery cost as illustrated in Fig. 2. Therefore, although the macro-cells are less loaded, the overall delivery cost is higher. The reason for this is that a hot-spot that is near the macrocell may have a high traffic demand that can be served with low air-time, and therefore a SBS will not be deployed in this site. This will translate to more data transmitted through the MBS core network, and therefore increase the delivery cost. In future work, we plan to investigate composite objective functions that consider the mutual effect of minimizing delivery cost and MBS resource consumption.

Finally, in Fig. 5 we illustrate the deployment success rates for the DPS MILPs and the DPS algorithms. Although we have seen that the algorithms have close to optimal performance, Fig. 5 indicates that they have a considerably lower success rate for medium values of N . This implies that there are cases with viable SBS deployment solutions which the algorithms are not able to generate. This is due to the limited search scope

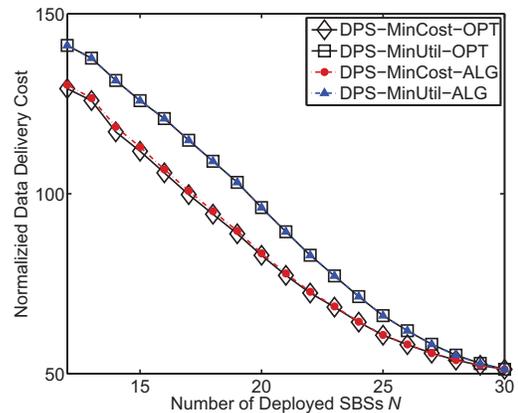


Fig. 2. Normalized delivery cost for varying SBS deployments.

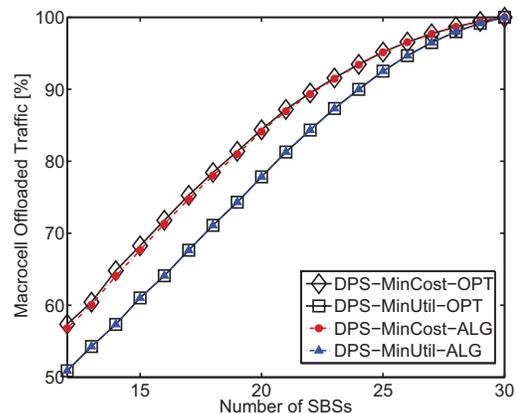


Fig. 3. Offloaded traffic percentage for varying SBS deployments.

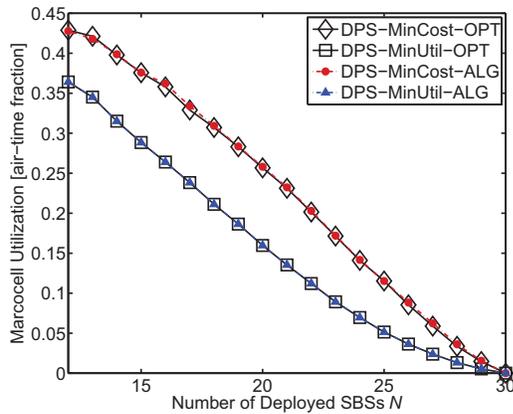


Fig. 4. Macrocell resource utilization for varying SBS deployments.

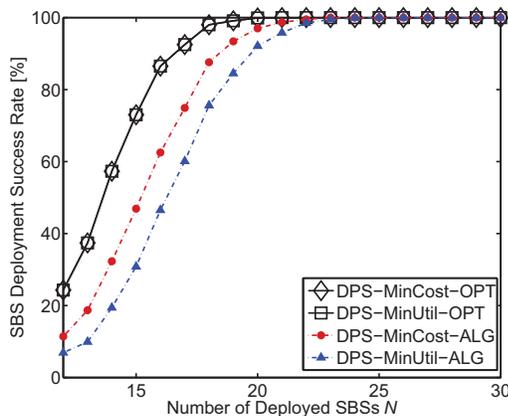


Fig. 5. Deployment success rate for varying SBS deployments.

of the algorithms to the violating macrocell. On the other hand, the DPS MILPs extend their search scope beyond the violating macrocells, even if this means having a deployment solution with a reduced performance. Therefore, another direction for future work is to implement additional iterative subroutines in the algorithms before ending the search procedure.

VI. CONCLUSION

In this paper, we investigated the small base station placement problem in high demand outdoor environments with the objectives of minimizing data delivery cost and macrocell resource utilization. We proposed two dynamic placement strategies that account for the resource and service constraints, and formulated them as MILP optimization problems. We then developed two greedy algorithms that achieve close to optimal performance albeit with a lower solution success rate. A detailed performance evaluation of the proposed deployment strategies was conducted, and the trade-offs between delivery cost and MBS utilization were discussed. Our results demonstrate that significant delivery cost and MBS utilization reductions are possible by incorporating the proposed SBS deployment strategies. At higher loads, it is recommended

to pursue a DPS that minimizes MBS utilization to allow the MBS to serve additional users if needed. On the other hand, a DPS that minimizes delivery cost is operationally more efficient at lower loads, without violating site demands. Future work includes investigating composite deployment objective functions and network power consumption reduction.

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