BER Performance of OFDM Systems in Mobile Multi-Hop Relaying Channels

Hassan A. Ahmed*, Ahmed Iyanda Sulyman•, Hossam Hassanein+
Electrical and Computer Engineering Department*•, School of Computing+
*+ Queen's University, Canada, Kingston, K7L 3N6
• King Saud University, Saudi Arabia, Riyadh 11421
Emali:{*3haea@queensu.ca, •asulyman@ksu.edu.sa, +hossam@cs.queensu.ca}

ABSTRACT

Orthogonal frequency division multiplexing (OFDM) system has been proposed as a technique for broadcasting digital signals and for wireless communication. The bit-error-rate (BER) performance of the OFDM system is severely affected by the nonlinearity of the high power amplifier and by the Doppler effect impairments. In this paper, we analyze the effects induced on the OFDM signal by the amplifier nonlinearity and by the Doppler effect over multi-hop relaying channels. Moreover, simulation results are presented to validate the analysis. It is shown that the resulting inter-carrier interference (ICI) due to the cumulative effects of the phase noise generated by these impairments per hop becomes very significant in a multi-hop relaying communication system, and severely degrades the BER performance of the system. Theoretical results show perfect agreement with those obtained by simulation.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: wireless communication; C.2.3 [**Computer-Communication Networks**]: Network Operations—*Network management*

General Terms

Theory, Design, Performance.

Keywords

Mobile Multi-hop relaying, WiMAX networks, mesh mode, OFDM systems, Amplify-and-forward relaying, Broadband wireless access.

1. INTRODUCTION

Multi-hop relaying has been adopted in several wireless networks such as 3G cellular, WLANs, and WiMAX systems as a cost-efective means of extending the reach or capacity of the wireless system. The multi-hop relay extension for IEEE 802.16e (mobile WiMAX) system, for instance, is the subject of ongoing standardization activities within the IEEE 802.16j Task Group. The emerging IEEE 802.16j standard enhances the IEEE 802.16e PHY and MAC to enable support of multi-hop communication between a mobile station (MS) and a base station (BS) through intermediate relay stations (RSs), which can be mobile or fixed. In such a system the communication between MS and BS is done through two hops: first hop between MS and RS and second hop between RS and BS. Each RS amplify or decode users data and forward it to the BS. Deploying RSs in the coverage area of BS as defined in IEEE 802.16j has been considered a promising solution that can replace the 802.16e mesh mode for coverage extension, throughput enhancement, and overcoming coverage holes.

Broadband wireless access networks (BWANs) based on orthogonal frequency division multiplexing (OFDM) have also gained tremendous attentions lately. OFDM provides an efficient broadband data transmission by sending parallel data over a number of closely-spaced subcarriers. For high data rate, it is desirable to increase the number of subcarriers per OFDM symbol. As the number of subcarriers increases for a fixed channel size however, the frequency spacing between the subcarriers in the OFDM symbol is reduced. This makes the OFDM system more sensitive to phase noise which destroys the orthogonality of the subcarriers, causing intercarrier interference (ICI).

The main issues causing phase noise in OFDM systems can be enumerated as follows. (i) High PAPR (peak-to-average power ratio) with nonlinear amplifier: In OFDM systems, because of high fluctuations in the level of the instantaneous signals transmitted, PAPR is typically large, making operation over linear region of high power amplifier (HPA) difficult. When some portions of the OFDM signals transmitted operate in the nonlinear region of the HPA, nonlinear distortions are introduced in the OFDM signals, resulting in phase noise which contributes to the ICI [1]. (ii) Doppler Effect: Relative speed between the transmitter and receiver in a wireless channel, introduces Doppler shifts in the received frequencies, contributing also to the ICI. However, these items still pose significant problems in most practical systems and should be given due considerations in the design of broadband multi-hop relaying communication systems, where the OFDM symbols typically traverse several hops from source to the destination nodes. The cascade effect of the multiple relaying channel dramatically amplifies the phase noise problem for the underlying OFDM system [2].

The ICI due to the phase noise is usually evaluated in terms of signal-to-interference ratio (SIR) or signal-to-interference-plus-noise ratio (SINR) [3], [4]. Therefore, this analysis is not easy mathematically, but the performance degradation is accurately characterized by bit-error-rate (BER) and symbol-error-rate (SER).

In [5], Keller and Hanzo apply the Gaussian approximation of the ICI to obtain expressions for BER analysis in additive white Gaussian noise (AWGN) channel. It has been shown by simulation that such an approximation is pessimistic when the BER is small. In [6], Zhao and Haggman proposed more accurate BER expression that uses the moments of the ICI distribution. Moreover, Sathananthan and Tellmbura in [7], derived the exact SER in AWGN channel. The disadvantage of the proposals in [5]- [7] is that the authors only consider the frequency shift effects in AWGN channel, while OFDM system is designed to overcome problems arise due to multipath channels. However all these works have been conducted in the context of single-hop communications.

In this paper, we perform an analytical analysis to study the effects of HPA nonlinearity ($\Phi_{Amp,r}$) and Doppler shift ($\Phi_{Dop,r}$) on the transmitted OFDM signal, and identify the statistical characteristics of these error terms over multi-hop relaying channels. We compute the probability of error in order to calculate the BER performance in the presence of HPA distortion and Doppler shift effect. We also examine by simulation the BER performance of different OFDM signals generated by *M-ary* quadrate amplitude modulation (*M-QAM*) constellation over multi-hop relaying channels and quantify the effects of Doppler shifts and HPA distortions as well as the cumulative effects of these impairments over multi-hop relaying channels. Simulation results of the OFDM signal distortion validate the analysis.

The remainder of this paper is organized as follows. Section II presents the OFDM system model with both amplifier and Doppler effects. In Section III, we derive the BER expressions for Rayleigh fading channel. In Section IV, we validate the theoretical analysis by means of simulation results. Finally, Section V concludes the paper.

2. SYSTEM MODEL

Consider a broadband wireless network employing OFDM transmission over multi-hop relaying channels, with each OFDM symbol consisting of N_c subcarriers. Consider the uplink scenario, where the transmitted signal from a mobile station (MS) originating the data (or source node, $\mathcal{S}$), passes through relay stations (RS) to the base station (BS) (or the destination node, $\mathcal{D}$). We assume that there are L users in the system, uniformly distributed in the coverage area of the BS, and that each user can be associated with the BS or a RS whichever provides stronger signal-to-noise ratio (SNR). An example of this model for the case $R = 2$ (two-hops relay network) is illustrated in Fig.1 for a cellular deployment, in which each cell is serviced by a BS, located at the center of the cell, and six RSs, each equidistant from the BS and located at the center of each side of the hexagon as shown.

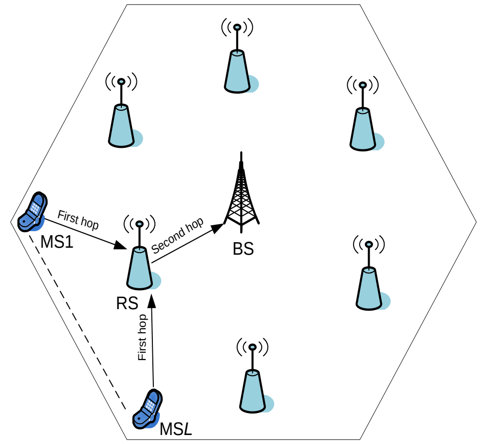


Figure 1: Mobile Multi-hop Relay System

We model a wireless relaying system where the transmitted OFDM symbol from an MS passes through multi-hop relaying channels $H_r, r = 0, 1, \ldots, R$ to the BS. Each transmitted OFDM symbol is effected by the channel impairments of each hop traverse through (i., e., $\Phi_{Amp,r}$, and $\Phi_{Dop,r}$). We assume that the Amplify-and-Forward (AF) relay option is employed at the relay nodes, where RSs simply amplify and froward the OFDM symbol at the radio frequency (RF) stage, without decoding its content. All relay nodes employ amplifiers with gain (or amplification factor) α_{ϕ}. The channel for each r^{th} hop transmission is modelled as a rayleigh fading channel, where $H_r[k]$ is the fading on the k^{th} subcarrier, $k = 0, 1, \ldots, N_c - 1$. These channels are modelled as independent and identically distributed (i.i.d) random variables. Fig.2 depicts the cascade effect of the multi-hop channel on the OFDM system. Each MS's data which in our case generated by quadrature amplitude modulation (QAM) is modulated with a number of N_c subcarreirs to produce N_c data points in frequency-domain $X[k]$. These data points are feed to an Inverse Fast Fourier Transform (IFFT) function whose output is time-domain signal $\widehat{X}(n)$ to transmit over a frequency Rayleigh fading channel after adding cyclic prefix (CP) to remove inter symbol interference (ISI), which introduced by the multipath channel in each hop [8].

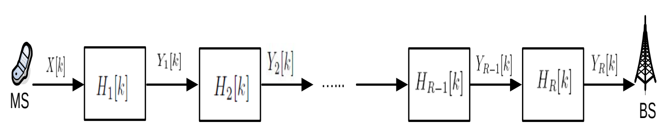


Figure 2: AF mobile multi-hop relaying channel model for OFDM systems

Therefore, at the receiver side after the cyclic prefix is removed, the time-domain received signal is feed to Fast Fourier Transform (FFT) function whose output is frequency-domain signal. Then the demodulation is performed to recover N_c data points. Where $Y_r[k]$, $r = 0, 1, \ldots, R$ denotes the received signal on the k^{th} subcarrier at the r^{th} hop. The received signal on the k^{th} subcarrier after passing through the first wireless relay hop, $r = 1$ is obtained in [8], and can be written as

$$Y_r[k] = H_r[k]X[k]\Phi_{Amp,r}[0]\Phi_{Dop,r}[0] + \beta_r[k] + W_r[k] \quad (1)$$

where $\Phi_{Amp,r}[0]$ and $\Phi_{Dop,r}[0]$ are the amplifier distortion and the Doppler shift effect impairments, respectively, $X[k]$ is the frequency-domain transmitted signal, $H_r[k]$ is the frequency-domain channel coefficient, $W_r[k]$ is the AWGN can be considered as a Gaussian random variable with zero mean and variance $\sigma^2_{W_r}$, and $\beta_r[k]$ is the ICI signal with variance $\sigma^2_{\beta_r}$ given by

$$\beta_r[k] = \Bigg(\sum_{m=0,m\neq k}^{N_c-1} X[m]H_r[m]\Phi_{Amp,r}[m-k] \Phi_{Dop,r}[m-k]\Bigg), \quad m = 0,1,\ldots,N_c-1 \quad (2)$$

The received signal on the k^{th} subcarrier after passing through two wireless relay hops, $R = 2$ can be expressed as

$$Y_r[k] = H_r[k]Y_{r-1}[k]\Phi_{Amp,r}[0]\Phi_{Dop,r}[0] + \beta_r[k] + W_r[k] \quad (3)$$

$Y_r[k]$ can be obtained by inserting the received signal on the k^{th} subcarrier after passing through $r-1$ hop, $Y_{r-1}[k]$ as follows

$$Y_r[k] = H_r[k]H_{r-1}[k]X[k]\Phi_{Amp,r-1}[0]\Phi_{Dop,r-1}[0] \Phi_{Amp,r}[0]\Phi_{Dop,r}[0] + H_r[k]\beta_{r-1}[k]\Phi_{Amp,r}[0] \Phi_{Dop,r}[0] + H_r[k]W_{r-1}[k]\Phi_{Amp,r}[0]\Phi_{Dop,r}[0] + \beta_r[k] + W_r[k] \quad (4)$$

3. BER ANALYSIS

In this section, we derive the BER performance of OFDM systems in multi-hop relaying channels employing QAM constellation signals over Rayleigh fading channels. The BER performance is derived in the presence of amplifier distortion $\Phi_{Amp,r}$ and Doppler shift $\Phi_{Dop,r}$ impairments. The received signal at the output of the second hop is obtained in section II, and presented in (4), therefore the SINR formula for two-hop network ($R = 2$) can be expressed as

$$SINR_r = \Bigg(E[|H_r[k]H_{r-1}[k]X[k]\Phi_{Amp,r}[0]\Phi_{Dop,r}[0] \Phi_{Amp,r-1}[0]\Phi_{Dop,r-1}[0]|^2]\Bigg) \Bigg/ \Bigg(E[|H_r[k] \beta_{r-1}[k]\Phi_{Amp,r}[0]\Phi_{Dop,r}[0] + H_r[k]W_{r-1}[k] \Phi_{Amp,r}[0]\Phi_{Dop,r}[0] + \beta_r[k] + W_r[k]|^2]\Bigg) \quad (5)$$

Without loss of generality, we consider the error rate for the zeroth subcarrier ($i., e., k = 0$), therefore our aim is to calculate for a given symbol sent on the zeroth subcarrier, the probability of error of the received symbol on that subcarrier after traverse through $r = 0,1,\ldots,R$ hops. We assume that the data symbols $X[k]$ and $X[m]$, $k = 0$, and $m = 1,\ldots,N_c-1$ are mutually independent random variables with average power E_s normalized to one. $H_r[k]$ and $H_{r-1}[k]$ are the frequency-domain channels coefficients for the first and second hop, respectively. $W_r[k]$ and $W_{r-1}[k]$ are the complex Gaussian noise with variance $\sigma^2_{W_r}$ and $\sigma^2_{W_{r-1}}$, respectively. $\beta_r[k]$ and $\beta_{r-1}[k]$ are the ICI terms caused by the loss of orthogonality between subcarriers in first and second hop, with variance $\sigma^2_{\beta_r}$ and $\sigma^2_{\beta_{r-1}}$, respectively. Assume that the phase rotation imposed by the channel, $\Phi_{Amp,r}[0]$ and $\Phi_{Dop,r}[0]$ can be accurately estimated at the r^{th} hop receiver, therefore (5) can be simplified as

$$SINR_r = \frac{E[H^2{}_r[k]]E[H^2{}_{r-1}[k]]}{E[H^2{}_r[k]]\sigma^2_{\beta_{r-1}} + E[H^2{}_r[k]]\sigma^2_{W_{r-1}} + \sigma^2_{\beta_r} + \sigma^2_{W_r}} \quad (6)$$

Let V_r be a random variable represents the multiplicative distortion imposed on the received signal of the k^{th} subcarrier when traverse through the r^{th} hop. The probability density function (PDF) of the Rayleigh distribution random variable V_r given by

$$f_{V_r}(v_r) = \frac{v_r}{\sigma^2(1-\lambda_r^2)} e^{\frac{-v_r^2}{2\sigma^2(1-\lambda_r^2)}} \quad (7)$$

where λ_r^2 is the correlation coefficient between $h_r(0)$ and $h_r(N_c-1)$. This coefficient given by [9]

$$\lambda_r^2 = \frac{E[v_n^2 v_k^2] - E[v_n^2]E[v_k^2]}{\sqrt{\{E[v_k^4] - E[v_k^2]^2\}\{E[v_n^4] - E[v_k^2]^2\}}} = J_0(2\pi f_{dr}|m-k|) \quad (8)$$

where $[.]$ is the expected value, f_{dr} is the maximum Doppler shift, and $J_0(.)$ is the zero order Bessel function of the first kind. Let a new Random Variable Z represents a product of two independent Rayleigh Random Variables (i., e., cascade channels) H_r, $r = 0,1$ expressed as [10]

$$Z = \prod_{r=1}^{R} V_r \quad (9)$$

We have two independent Rayleigh random variables V_r, and V_{r-1} in cascade, therefore

$$Z = V_r V_{r-1} \quad (10)$$

The PDF of the resulting random variable Z can be expressed as [11]

$$f_Z(z) = 2K_0(2\sqrt{Z}) \quad (11)$$

where $K_0(.)$ is the modified Bessel function of the second kind and zero order. For large number of N_c, we invoke the central limit theory to assume that the ICI terms $\beta_r[k]$ and $\beta_{r-1}[k]$ are approximated by Gaussian Random Variables with zero mean. The total power of ICI is the total power the desired subcarrier receives from the other subcarriers, hence the variance of the ICI can be obtained by the difference between the total power of ICI and the desired signal power. The desired signal power of the k^{th} subcarrier is given by $E[|ZX[k]|^2] = E_s E[Z^2]$, where E_s is the energy for the transmitted QAM symbol on subcarrier k normalized to one. The expectation $E[Z^2]$ can be derived as follow

$$E[V_{r-1}^2] = E\left[\left(\frac{1}{N_c}(\sum_{n=0}^{N_c-1}|h_{r-1}(k)|)(\sum_{k=0}^{N_c-1}|h_{r-1}(l)|)\right)\right], \quad k = l = 0,\ldots,N_c-1 \tag{12}$$

$$E[V_r^2] = E\left[\left(\frac{1}{N_c}(\sum_{n=0}^{N_c-1}|h_r(m)|)(\frac{1}{N_c}\sum_{k=0}^{N_c-1}|h_r(n)|)\right)\right], \quad m = n = 0,\ldots,N_c-1 \tag{13}$$

Therefore,

$$E[Z^2] = E[V_{r-1}^2]E[V_r^2] \tag{14}$$

$$E[Z^2] = \frac{1}{N_c^4}\sum_{k=0}^{N_c-1}\sum_{l=0}^{N_c-1}\sum_{m=0}^{N_c-1}\sum_{n=0}^{N_c-1}\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty v_{r-1}(k)v_{r-1}(l)v_r(m)v_r(n)f(v_{r-1}(k),v_{r-1}(l)) f(v_r(m),v_r(n))dv_{r-1}(k)dv_{r-1}(l)dv_r(m)dv_r(n) \tag{15}$$

Substituting the PDF of V_{r-1} and V_r, and perform the integrations using eqs. (8.406.3), (6.631.1) and eqs. (7.621.4), (9.131.1) in [12], therefore eq. (15) can be simplified as

$$E[Z^2] = \frac{1}{N_c^4}\Bigg[2N_c + \pi\sum_{i=1}^{N_c-1}(N_c-i)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;\lambda_{r-1}^2\right) + 2N_c + \pi\sum_{i=1}^{N_c-1}(N_c-i).{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;\lambda_r^2\right)\Bigg] \tag{16}$$

Eq.(16) can be rearranged as

$$E[Z^2] = \Bigg[\frac{2}{N_c^3} + \frac{\pi}{N_c^4}\sum_{i=1}^{N_c-1}(N_c-i)\left(.{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;\lambda_{r-1}^2\right) + {}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;\lambda_r^2\right)\right)\Bigg] \tag{17}$$

Where $F(.,.;.;.)$ is the hypergeometric function. The variance of the ICI of the k^{th} subcarrier at the r^{th} hop can be derived by subtracting the desired signal power from the total power of ICI [13] as follows

$$\sigma_{\beta_{r-1}}^2 = \left(E\left[\frac{1}{N_c}\sum_{l=0}^{N_c-1}|h_{r-1}(l)|\right] - E[V_{r-1}^2]\right) \tag{18}$$

Inserting $E[V_{r-1}^2]$ from (12) into (18) gives

$$\sigma_{\beta_{r-1}}^2 = \int_0^\infty \frac{v_{r-1}(l)}{\sigma^2}e^{-\frac{v_{r-1}^2}{2\sigma^2}}dv_{r-1}(l) - \frac{1}{N_c^2}\sum_{k=0}^{N_c-1}\sum_{l=0}^{N_c-1}\int_0^\infty\int_0^\infty v_{r-1}(k)v_{r-1}(l)f(v_{r-1}(k),v_{r-1}(l)) dv_{r-1}(k)dv_{r-1}(l) \tag{19}$$

This yields to

$$\sigma_{\beta_{r-1}}^2 = \left(1 - \frac{1}{N_c^2}\left[2N_c + \pi\sum_{i=1}^{N_c-1}(N_c-i)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;\lambda_{r-1}^2\right)\right]\right) \tag{20}$$

Similarly, the variance of ICI term in the second hop $\sigma_{\beta_r}^2$ can be obtained as follows

$$\sigma_{\beta_r}^2 = \left(E\left[\frac{1}{N_c}\sum_{n=0}^{N_c-1}|h_r(n)|\right] - E[V_r^2]\right) \tag{21}$$

Substituting $E[V_r^2]$ from (13) into (21) gives

$$\sigma_{\beta_r}^2 = \int_0^\infty \frac{v_r(n)}{\sigma^2}e^{-\frac{v_r^2}{2\sigma^2}}dv_r(n) - \frac{1}{N_c^2}\sum_{m=0}^{N_c-1}\sum_{n=0}^{N_c-1}\int_0^\infty\int_0^\infty v_r(m)v_r(n)f(v_r(m),v_r(n)) dv_r(m)dv_r(n) \tag{22}$$

This can be expressed as

$$\sigma_{\beta_r}^2 = \left(1 - \frac{1}{N_c^2}\left[2N_c + \pi\sum_{i=1}^{N_c-1}(N_c-i)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;\lambda_r^2\right)\right]\right) \tag{23}$$

Furthermore, we can obtain the $E[|H_r[k]\beta_{r-1}[k]|^2]$ term in (6) as follows

$$E[|H_r[k]\beta_{r-1}[k]|^2] = E[|H_r[k]|^2]\sigma_{\beta_{r-1}}^2 \tag{24}$$

We have $E[|H_r[k]|^2] = E[|V_r|^2]$, therefore

$$E[|H_r[k]\beta_{r-1}[k]|^2] = E[|V_r|^2]\sigma_{\beta_{r-1}}^2 \tag{25}$$

Therefore substituting $E[|V_r|^2]$ into (25) gives

$$E[|H_r[k]\beta_{r-1}[k]|^2] = \frac{1}{N_c^2}\Bigg[2N_c + \pi\sum_{i=1}^{N_c-1}(N_c-i)\, {}_2F_1\Big(-\frac{1}{2},-\frac{1}{2};1;\lambda_r^2\Big)\Bigg] \Bigg(1-\frac{1}{N_c^2}\Bigg[2N_c+\pi\sum_{i=1}^{N_c-1}(N_c-i)\, {}_2F_1\Big(-\frac{1}{2},-\frac{1}{2};1;\lambda_{r-1}^2\Big)\Bigg]\Bigg) \quad (26)$$

and the $E[|H_r[k]W_{r-1}[k]|^2]$ term in (6) can be expressed as follow

$$E[|H_r[k]W_{r-1}[k]|^2] = \frac{\sigma^2_{W_{r-1}}}{N_c^2}\Bigg[2N_c+\pi\sum_{i=1}^{N_c-1}(N_c-i)\, {}_2F_1\Big(-\frac{1}{2},-\frac{1}{2};1;\lambda_r^2\Big)\Bigg] \quad (27)$$

Substituting $E[Z^2]$, $\sigma^2_{\beta_{r-1}}$ and $\sigma^2_{\beta_r}$, from (17), (20) and (23), respectively, and terms from (26) and (27) into (6) gives

$$SINR_r = \frac{E[Z^2]}{E[V_r^2]\sigma^2_{\beta_{r-1}} + E[V_r^2]\sigma^2_{W_{r-1}} + \sigma^2_{\beta_r} + \sigma^2_{W_r}} \quad (28)$$

When there are two hops in the network and M-QAM modulation is used, the probability of error per subcarrier k can be expressed as [14]

$$P_e(E,k) = \int_0^\infty \Bigg\{1-\Bigg[1-\frac{2(\sqrt{M}-1)}{\sqrt{M}} Q\Big(\sqrt{2gSINR_r}\Big)\Bigg]^2\Bigg\} f_Z(z)dZ \quad (29)$$

Where M is the number of constellation points in the modulation scheme, $Q(x)$ is the Gaussian Q-function defined as $Q(x) = (1/\sqrt{2\pi})\int_x^\infty e^{-t^2/2}dt$, and g is a constant depending only on the modulation scheme given by $g = \frac{3}{(2(M-1))}$. Using the probability of error $P_e(E,k)$ obtained above, the average OFDM symbol error rate can be expressed as

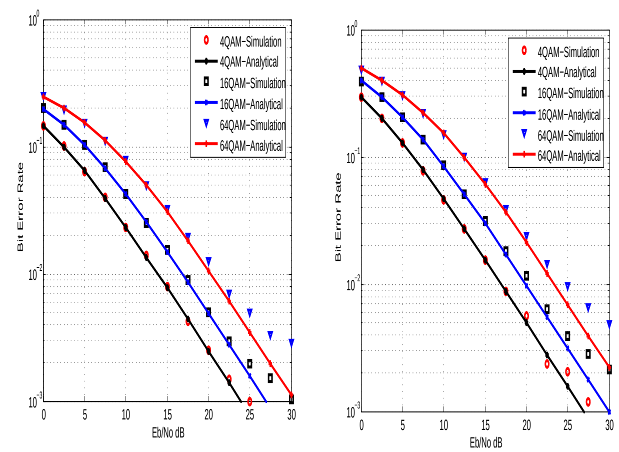
$$SER_{OFDM} = \frac{1}{N_c}\sum_{k=0}^{N_c-1} P_e(E,k) \quad (30)$$

Finally, the bit error rate for subcarrier k can be calculated using the SER_{OFDM} as follow

$$BER = \frac{SER_{OFDM}}{log_2(M)} \quad (31)$$

4. SIMULATION RESULTS

In this section, we present simulation results to verify the BER analysis of 256-OFDM system in multi-hop relaying channel. The BER performance is evaluated in the presence of both the HPA distortions and Doppler effect impairments. In the simulation model, three signal constellations are used 4-QAM, 16-QAM, and 64-QAM. The CP is added to each symbol to overcome ISI effects, the CP can take the values 25%, 12.5%, 6.25%, and 3.125% of the OFDM symbol length. However, in this work only CP = 25% is used. The Doppler shift values used in the simulation are 50 Hz, and 100Hz, while the HPA distortion values are $\pi/12$, and $\pi/6$, respectively. The transmission of 25000 OFDM symbols through $R = 1$ and $R = 2$ hops is simulated, then the BER performance for subcarrier k is measured.
Fig.3(a) presents the BER performance of single-hop OFDM system in the presence of Doppler effect and HPA nonlinearity distortion. For this illustration we used the following values $f_d = 50Hz$, and $\alpha_\phi = \pi/12$. It can be observed that the error floor starts to appear at higher E_b/N_0 due to ICI component as a result of Doppler effect and HPA distortion. Furthermore, as the constellation order M increases, the ICI distortion becomes unacceptably large.



(a) $R = 1$, $f_d = 50Hz$, $and \alpha_\phi = \pi/12$

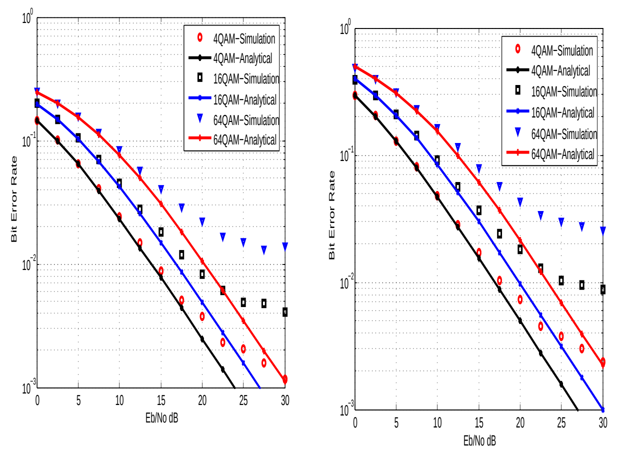
(b) $R = 2$, $f_d = 50Hz$, $and \alpha_\phi = \pi/12$

Figure 3: BER versus E_b/N_0 of OFDM system over Rayleigh Fading channel

In Fig.3(b), we display the BER performance of two-hop OFDM system in the presence of Doppler effect and HPA nonlinearity distortion. For this illustration we used the following values $f_d = 50Hz$, and $\alpha_\phi = \pi/12$. From the curves, It can be observed that the error floor starts to appear at lower E_b/N_0 comparing to the single-hop case (Fig. (3a)) due to ICI component as a result of Doppler effect and HPA distortion as well as their cumulative effects over multi-hop relaying channels. Similar to single-hop as the constellation order M increases, the ICI distortion becomes more annoying. In this case, ICI distortion governors the system performance improvement.

Fig.4(a) shows the BER performance of single-hop OFDM system. For this demonstration we used the following values $f_d = 100Hz$, and $\alpha_\phi = \pi/6$. It can be noticed that, the error floor in the case of 16 QAM constellation is higher then the error floor in the case of 4 QAM constellation. While the highest error floor is measured for 64QAM constellation case. Thus, as the constellation order M increases, the ICI distortion increases.

Fig.4(b) illustrates the BER performance of two-hop OFDM system. For this illustration we used the following values $f_d = 100Hz$, and $\alpha_\phi = \pi/6$. It can be observed that the error floor in the case of 16 QAM constellation and multi-hop system is higher than that in the case of single-hop for the same constellation order. The highest measured error floor is in the case of multi-hop and 64 QAM constellation order due to ICI component as a result of Doppler effect and HPA distortion and their cumulative effects over multi-hop relaying channels. Hence, the number of hops and the constellation order control the performance of the BER in the system.



(a) $R = 1, f_d = 100Hz, and \alpha_\phi = \pi/6$

(b) $R = 2, f_d = 100Hz, and \alpha_\phi = \pi/6$

Figure 4: BER versus E_b/N_0 of OFDM system over Rayleigh Fading channel

5. CONCLUSION

An analytical analysis of the effects of amplifier nonlinearity and Doppler effect on the OFDM system has been presented in this paper. The HPA shifts the transmitted signal before transmission over the channel, resulting in ICI. Also, the relative speed between the transmitter and the receiver shifts the transmitted frequency at the receiver side causing also ICI. The resulting ICI due to the cumulative effects of the amplifier non-linearity and Doppler shift per hop becomes very significant in IEEE 802.16j multi-hop relaying communication system. Analytical BER performance of the system is evaluated and simulation results are presented to verify the analysis. Based on some observations from the BER curves, the following conclusions can be drawn. (i) OFDM system is sensitive to ICI and this sensitivity is increased with the constellation order M. (ii) Amplifier distortion is one cause of the ICI, which depends on the amplifier characteristics. (iii) The Doppler effect is another cause of the ICI, which increases with the speed of the MS. (iv) The resulting ICI due to the above cumulative effects per hop becomes more annoying over multi-hop channels. The aforementioned issues should be given due consideration in order to improve the BER performance in IEEE 802.16j mobile multi-hop relaying system.

6. ACKNOWLEDGMENTS

This research is supported in part by the Natural Science and Engineering Research Council of Canada (NSERC) and by a grant from Qatar Telecom (QTel). The first author would like to thank the Ministry of Higher Education in Libya for the financial support.

7. REFERENCES

[1] E. Costa, and S. Pupolin, “M-QAM-OFDM System Performance in the Presence of a Nonlinear Amplifier and Phase Noise,” *IEEE Transac. on commun.*, vol. 50, no. 3, pp. 462 - 472, March. 2002.

[2] A. I. Sulyman, G. Takahara, H. S. Hassanein, and M. Kousa, “Multihop Capacity of MIMO-Multiplexing Relaying Systems,” *IEEE Trans. on Wireless Commun.*, vol. 8, no. 6, 3095 - 3103, Jun. 2009.

[3] T. Pollet, M. Van Bladel, M. Moeneclaey,“BER Sensitivity of OFDM Systems to Carrier Frequency Offset and Wiener Phase Noise,” *IEEE Trans. on Commun.*, vol. 43, no. 2/3/4, pp.191 - 193, Apr. 1995.

[4] P. Liu, Y. Bar-Ness,“Closed Expressions for BER performance in OFDM Syestems with Phase Noise,” *IEEE-Intrnational Confrence on Communications*, pp.5366 - 5370, 2006.

[5] T. Keller and L. Hanzon “Adaptive multicarrier modulation: A convenient framework for time-frequency processing in wireless communication,” *IEEE*, vol. 88, no. 5, pp. 611 - 640, May 2000.

[6] Y. Zhao and S. G. Haggman“BER analysis of OFDM communication systems with intercarrier interference,” *IEEE Int. Conf. Communication Technology*, vol. 2, pp. 1-5 , Oct 1998.

[7] K. Sathananthan and C. Tellambura,“Probability of error calculation of OFDM systems with frequency offset,” *IEEE Trans. on Commun.*, vol. 49, no. 11, pp. 1884-1888 , Nov. 2001.

[8] H. Ahmed, A. Sulyman, H. Hassanein, “BER Performance of OFDM Systems with Channel Impairments,” *IEEE Local Computer Networks*, pp. 1027 - 1032, Oct. 2009.

[9] W. C. Jakes, Jr., “Microwave Mobile Communications,” *New York: Wiley*, 1974.

[10] J. Salo, H. El-Sallabi, “The Distribution of the Product of Independent Rayleigh Random Variables,” *IEEE Trans. Antennas and Propagation*, vol. 54, no. 2, Feb. 2006.

[11] V. Erceg, S. Fortune, J. Ling, A. Rustako, and R. Valenzuela, “Comparisons of a Computer-Based Propagation Prediction Tool with Experimental Data Collected in Urban Microcelluar Environments,” *IEEE Trans. on Commun.*, vol. 15, no. 4, pp. 677 - 684, May 1997.

[12] I. S. Gradshteyn and I. M. Ryzhik, “Table of integrals, series, and products,” 6th edition, *San Diego: CA: Academic Press*, 2000.

[13] G. L. Stuber, “Principles of Mobile Communication,” *Boston: Kluwer Academic*, 1996.

[14] R. Yen, H. Liu, W. Tsai,“QAM Symbol Error Rate in OFDM Systems Over Frequency-Selective Fast Ricean-Fading Channels,” *IEEE Trans. Vehc. Tech.*, vol. 57, no. 2, pp. 1322 - 1325, March. 2008.