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RESEARCH ARTICLE

Bit error rate performance of orthogonal frequency-division multiplexing relaying systems with high power amplifiers and Doppler effects

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ABSTRACT

This paper analyzes the bit error rate performance of orthogonal frequency-division multiplexing systems in mobile multihop relaying channels. We considered the uplink scenario and quantified the effects of mobile channel impairments such as Doppler shift due to user mobility per hop, high power amplifier distortions when amplifying the transmitted/relayed orthogonal frequency-division multiplexing symbol per hop, and the cumulative effects of these impairments on multihop relaying channels. It was shown that the resulting intercarrier interference due to the cumulative effects of the phase noise generated by these impairments per hop becomes very significant in a multihop relaying communication system and severely degrades the bit error rate performance of the system. Simulation results agree well with and validate the analysis. Copyright © 2011 John Wiley & Sons, Ltd.

KEYWORDS
mobile multihop relaying; WiMAX networks; OFDM systems; amplify-and-forward relaying; broadband wireless access

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1. INTRODUCTION

Broadband wireless access networks (BWANs) such as Long Term Evolution (LTE) [1] and the Worldwide Interoperability for Microwave Access (WiMAX) [2,3] have gained tremendous attention lately for leveraging the support of a wide range of applications with different quality of service requirements. Despite the support for such range of applications, satisfying the different quality of service requirements while maximizing the network capacity and extending the network coverage is still a major issue in these networks. Multihop relaying has been adopted in several BWANs such as LTE and WiMAX as a cost-effective means of extending the reach and/or capacity of these wireless networks. The emerging multihop relaying extension enhances the conventional BWANs to enable support for multihop communication between a mobile station (MS) and a base station (BS) through intermediate relay stations (RSs). Deploying RSs in the coverage area of BS as defined in IEEE 802.16j has been considered a promising solution that can replace the 802.16e mesh mode for coverage extension, throughput enhancement, and overcoming coverage holes.

Orthogonal frequency-division multiplexing (OFDM) is the de facto transmission mechanism for next generation BWANs. OFDM provides an efficient broadband data transmission by sending parallel data over a number of closely spaced subcarriers. For high data rate, it is desirable to increase the number of subcarriers per OFDM symbol. As the number of subcarriers increases for a fixed channel size however, the frequency spacing between the subcarriers in the OFDM symbol reduces. This makes the OFDM system more sensitive to phase noise, which destroys the orthogonality of the subcarriers, causing intercarrier interference (ICI).

The main issues causing phase noise in OFDM systems can be enumerated as follows. (i) IQ imbalance: imbalance in the separate implementation of the in-phase (I) and the quadrature phase (Q) components of the OFDM signals can introduce some phase noise, contributing some ICI. (ii) Imperfect synchronization in local oscillators (LO): mismatch between the LO at the transmitter and the receiver introduces frequency offsets, which also contribute to the ICI. (iii) High peak-to-average power ratio (PAPR) with nonlinear amplifier: in OFDM systems, because of the high fluctuations in the level of the instantaneous signals transmitted, PAPR in OFDM systems is typically large, making operation solely over the linear region of a high power amplifier (HPA) difficult. When some portions of the OFDM signals transmitted operate in the nonlinear region of the HPA, nonlinear distortions are introduced in the OFDM signals, resulting in phase noise that also contributes to the ICI [4,5]. (iv) Doppler shift: relative speed between the transmitter and the receiver in a wireless channel introduces Doppler shift in the received frequencies, contributing also to the ICI. With a digital implementation of the IQ modulation circuits, IQ imbalance problems can be solved [5]. Also, in most practical systems, several high-performance classical frequency offset estimators for oscillator synchronization problems in OFDM systems are available [6–10]. However, items (iii) and (iv) still pose significant problems in most practical systems and should be given due considerations in the design of broadband multihop relaying communication systems, where the OFDM symbols typically traverse several hops from source to destination nodes. The cascade effect of the multiple relaying channel dramatically amplifies the phase noise problem for the underlying OFDM system [11].

To analyze the performance of an OFDM relaying system, effects such as HPA nonlinearity and Doppler shift should be taken into consideration. The high PAPR in OFDM systems is typically large, making operation over the linear region of the HPA difficult. However, the HPA effects on OFDM symbol have received little research attention. There are some studies reported in the literature that take into account the effect of power amplifier [12–14]. In [12], the effect of the HPA is avoided by considering sufficient back-off period; therefore, the power amplifier is considered to be linear. In [13], the receiver cancelation of power amplifier nonlinearity in a two-hop network was considered, whereas a detection technique for clipped OFDM symbols in a cooperative relay network was proposed in [14]. The downlink communication was considered and the bit error probability of OFDM relay links with nonlinear power amplifier was analyzed in [15].

Doppler shift arising from MS's mobility destroys the subcarriers' orthogonality, causing ICI; however, a number of works have been reported in the literature [16,17] for OFDM system performance analysis that are considering Doppler shift. In [16], an algorithm was proposed for Doppler shift estimation, but this algorithm can only be applied to signals on the pilot subcarriers. Therefore, to obtain an accurate estimation of Doppler shift, a large number of OFDM symbols are required, and this results in a longer estimation delay. In [17], an algorithm was proposed for Doppler shift estimation based on the autocorrelation of time-domain channel. However, the receiver still has to know the fading channel coefficient in order to perform the Doppler shift estimation. This process results in higher complexity at the receiver. A closed form of the ICI was derived in [18], which was used to determine the probability density function of the ICI. The analysis of ICI in terms of signal-to-interference ratio and signal-to-interference-plus-noise ratio (SINR) is reported in [19–21]. The performance of OFDM system was analyzed, considering the assumption that the ICI follows the Gaussian distribution [22–28]. In [26], the Gaussian approximation of the ICI was applied to obtain expressions for BER analysis in an additive white Gaussian noise (AWGN) channel. It was shown by simulation that such an approximation is pessimistic when the BER is small. A more accurate BER expression that uses the moments of the ICI distribution was proposed in [27]. Moreover, in [28], the exact symbol error rate (SER) in an AWGN channel was derived. In [26–28] however, the authors only considered the frequency shift effects on an AWGN channel, while the OFDM system is designed to overcome problems arising because of multipath fading channels. BER analysis using Gaussian approximation of the ICI in Rayleigh fading channel is reported in [29]. However, most of these works have been done only in the context of single-hop communications. The extent to which OFDM subcarriers lose orthogonality and consequently the resulting ICI when transmitting OFDM symbols over multihop relaying channel with the HPA nonlinearity and Doppler shift effects is to date not known for various OFDM configurations. Accurate characterization of the ICI problem due to Doppler shift and HPA distortion on multihop channel would allow the development of novel algorithms for alleviating the problem.

In this paper, we develop an analytical method to study the effects of HPA nonlinearity and Doppler shift on transmitted OFDM signals and present the statistical characteristics of these impairments on two-hop relaying channels. The main contribution of this paper is the derivation of the SINR expression as a function of the HPA nonlinearity and the Doppler effect. The obtained SINR expression was used to derive a closed-form formula for the probability of error that was used to calculate the average SER as well as measure the BER performance in the two-hop relaying OFDM system. In this paper, we also provide simulation results to validate the analysis.

The remainder of this paper is organized as follows. Section 2 presents the OFDM system model, taking into consideration the amplifier distortions and Doppler shift effects on two-hop relaying network, and the derivation of the end-to-end SINR expression. Section 3 demonstrates the BER performance analysis. In Section 4, we validate the theoretical analysis by simulation results, and finally, Section 5 concludes the paper.

2. SYSTEM MODEL

Consider a two-hop amplify-and-forward OFDM relaying system, with each OFDM symbol consisting of N_c

subcarriers. We consider the uplink scenario, where the transmitted signal from a mobile station (MS), where the data are originating (or source node, $\mathcal{S}$), passes through R relay stations (RSs) to the base station (BS) (or the destination node, $\mathcal{D}$). The direct channel between the source and destination is considered to be very attenuated and unable to provide a communication with acceptable quality. We assume that there are L users in the system, uniformly distributed in the coverage area of the BS, and that each user can be associated with the BS or an RS, whichever provides stronger signal-to-noise ratio (SNR). An example of this model for the case $R = 2$ (two-hop relay network), $r = 1, \ldots, R$, is illustrated in Figure 1 for a cellular deployment, in which each cell is serviced by a BS, located at the center of the cell, and six fixed RSs ($N = 6$), each equidistant from the BS and located at the center of each side of the hexagon as shown. We model a wireless relaying system where the transmitted OFDM symbol from an MS passes through two-hop relaying channels $H_r, r = 1, \ldots, R$, to the BS. Each transmitted OFDM symbol is affected by the channel impairments of each hop traversed.

Here, we focus on two impairments, namely, amplifier distortion (Φ_{Amp}) and Doppler shift (Φ_{Dop}). We assume that the amplify-and-forward relay option is employed at the relay nodes, where RSs simply amplify and forward the OFDM symbol at the radio frequency stage, without decoding its content. The RS demodulates the OFDM symbol and then amplifies all subcarriers separately. All relay nodes employ amplifiers with gain (or amplification factor) α. The mobile channel for the first-hop transmission (MS–RS) is modeled as a Rayleigh fading channel, where $H_{r-1}[k]$ is the fading on the kth OFDM subcarrier, $k = 0, 1, \ldots, N_c - 1$; therefore, the SINR of the first hop can be considered as an exponential random variable. The average SINR, $\overline{\gamma}_{r-1}[k] = E[\gamma_{r-1}[k]]$, can be different for all k. In the second hop, the source and destination are fixed infrastructures (relay station and base station); therefore, the (RS–BS) channel $H_r[k]$ can be considered as a static.[†] Thus, $\gamma_r[k] = \overline{\gamma}_r[k]$ for all k. Figure 2 depicts the cascade effect of the two-hop relaying channel on the OFDM system.

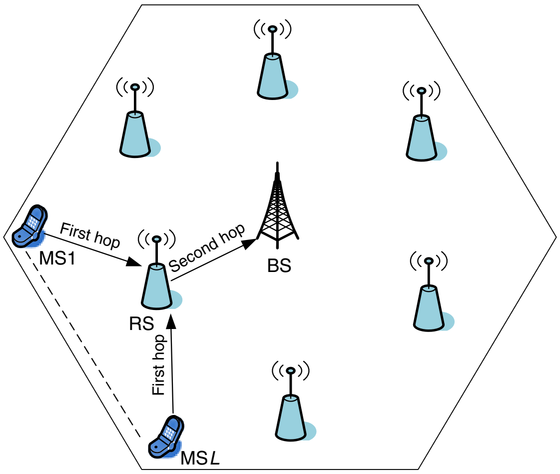


Figure 1. Mobile multihop relaying system. MS, mobile station; RS, relay station; BS, base station.

Each MS's data, which are considered in this system to be quadrature amplitude modulation (QAM) signal, are modulated with N_c subcarriers to produce N_c data points in frequency-domain $X[k]$, where the transmit power is $P = E[|X[k]|^2]$ for all k. These data points are fed to an inverse fast Fourier transform (IFFT) function whose output is a time-domain signal $x(\tau)$. We assume that the cyclic prefix (CP)[‡] is added to the OFDM symbol to combat intersymbol interference introduced by the channel in each hop. The time-domain signal $x(\tau)$ is fed to an HPA that is modeled by a static memoryless nonlinearity, $f(\cdot)$, to produce an amplified time-domain signal $\widehat{x}(\tau)$ expressed as

$$\widehat{x}(\tau) = x(\tau) + \phi_{\text{Amp}}(\tau) \tag{1}$$

where $\phi_{\text{Amp}}(\tau)$ is the signal distortion. The time-domain signal $\widehat{x}(\tau)$ can be written in the frequency domain as

$$\widehat{X}[k] = X[k] + \Phi_{\text{Amp}}[k] \tag{2}$$

When the HPA operates close to the saturation point, the distortion term $\mathit{\Phi}_{\text{Amp}}[k]$ becomes Gaussian [30], with power $\sigma^2_{\Phi_{\text{Amp}}} = E[|\Phi_{\text{Amp}}[k]|^2]$. For the second impairment, we assume that the transmitted signal is shifted by Doppler effect $\phi_{\text{Dop}}[k]$ after passing through the channel $H[k]$, with f_d denoting the frequency offset between the transmitter and the receiver and is given by $f_d = (f_c * V_{\text{SD}})/\text{c} = (V_{\text{SD}}/\lambda)$, where f_c is the transmitted frequency, V_{SD} is the velocity of the transmitter relative to the receiver (m/s), c is the speed of the wave (3×10^8 m/s), and $\lambda = \text{c}/f_c$ is the wavelength. The distortion term $\Phi_{\text{Dop}}[k]$ has power $\sigma^2_{\Phi_{\text{Dop}}} = E[|\Phi_{\text{Dop}}[k]|^2]$. Therefore, the received signal $Y_{\text{R}}[k]$ can be expressed as

$$Y_{\text{R}}[k] = \widehat{X}[k]H_{r-1}[k] + W_{\text{R}}[k] \tag{3}$$

where $W_{\text{R}}[k]$ is the AWGN term at the RS with power $\sigma^2_{\text{R}} = E[|W_{\text{R}}[k]|^2]$. Considering the frequency-domain signal at the HPA output $\widehat{X}[k]$ and Doppler shift effect $\Phi_{\text{Dop}}[k]$, Equation (3) can be rewritten as

$$\begin{aligned} Y_{\text{R}}[k] &= X[k]H_{r-1}[k] + \Phi_{\text{Amp,S}}[k]\Phi_{\text{Dop,SR}}[k]H_{r-1}[k] \\ &\quad + W_{\text{R}}[k] \end{aligned} \tag{4}$$

[†] Some channel fluctuations will be present because of mobile terminals in the propagation path between a source and a destination, but the static components will have stronger effects.

[‡] To simplify our analysis, the cyclic prefix was not directly reflected in our equations, but its use is implied.

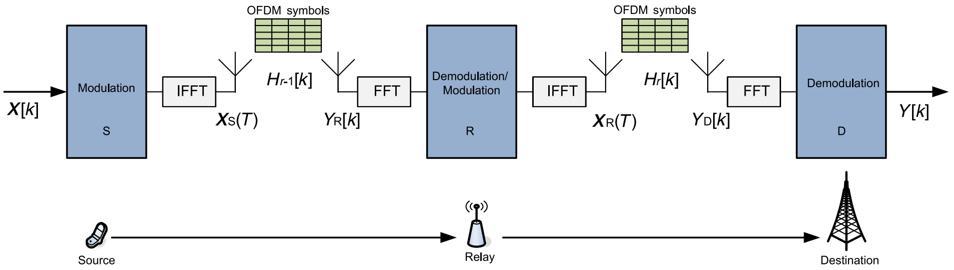


Figure 2. Two-hop amplify-and-forward relaying channel model for orthogonal frequency-division multiplexing (OFDM) systems. FFT, fast Fourier transform; IFFT, inverse FFT.

where $\Phi_{\text{Amp,S}}[k]$ is the HPA effect on the source (MS) and $\Phi_{\text{Dop,SR}}[k]$ is the Doppler shift effect on the first hop. Figure 3 demonstrates the ICI effect on an OFDM symbol with seven subcarriers.

Therefore, at the receiver side after the CP is removed, the time-domain received signal is fed to fast Fourier transform (FFT) function whose output is a frequency-domain signal. Then, the demodulation is performed to recover the QAM signals on each of the N_c subcarriers. The RS amplifies each subcarrier with amplification factor $\alpha[k]$, where $X_R[k] = \alpha[k]Y_R[k]$. The well-known nonlinear memoryless amplifier [31] is adopted in this paper, and the expression for $\alpha[k]$ is given by

$$\alpha[k] = \sqrt{\frac{P_R}{(\sigma^2_{\Phi_{\text{Amp,S}}})P_S|H_{r-1}[k]|^2 + \sigma^2_R}} \tag{5}$$

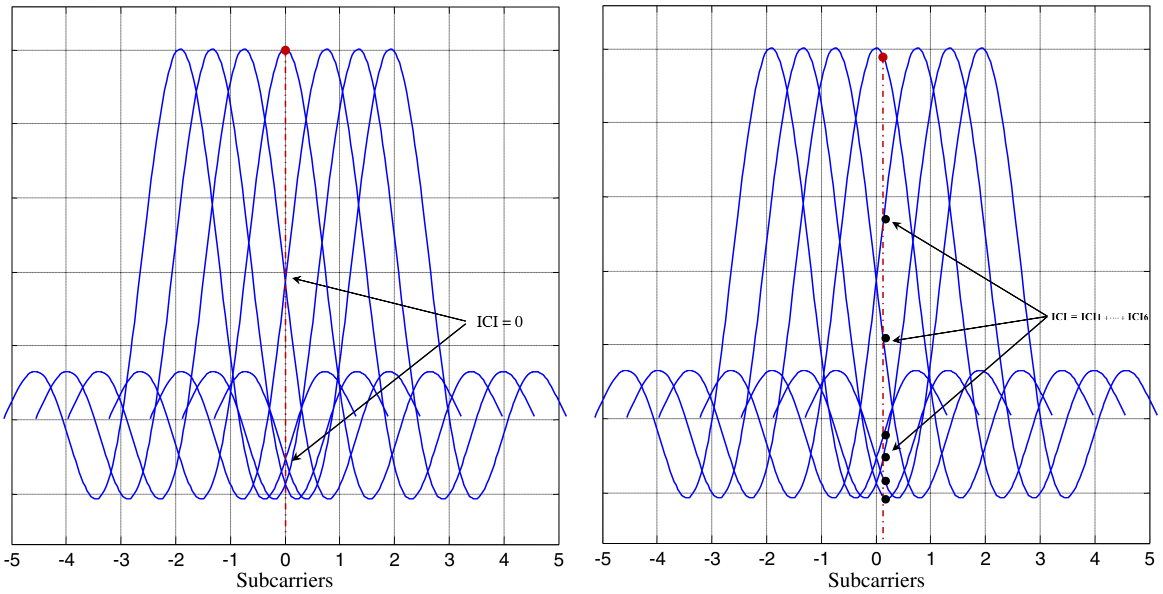
Therefore, the signal at the RS's HPA output can be simplified as

$$\begin{aligned} X_R[k] &= \alpha[k]X[k]H_{r-1}[k] \\ &\quad + \alpha[k]\Phi_{\text{Amp,S}}[k]\Phi_{\text{Dop,SR}}[k]H_{r-1}[k] \\ &\quad + \alpha[k]W_R[k] \end{aligned} \tag{6}$$

After the amplified signal $X_R[k]$ is transmitted over the channel of the second-hop $H_r[k]$, the received signal $Y_D[k]$ can be expressed as

$$Y_D[k] = X_R[k]H_r[k] + \Phi_{\text{Amp,R}}[k]\Phi_{\text{Dop,RD}}[k] + W_D[k] \tag{7}$$

where $W_D[k]$ is the AWGN term at the destination with power $\sigma^2_D = E[|W_D[k]|^2]$, $\Phi_{\text{Amp,R}}[k]$ is the HPA effect on



(a) OFDM symbol without frequency shift

(b) OFDM symbol with frequency shift

Figure 3. Intercarrier interference (ICI) effects on orthogonal frequency-division multiplexing (OFDM) symbol.

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the relay (RS), and $\Phi_{\mathrm{Dop,RD}}[k]$ is the Doppler shift effect on the second hop. Substituting the amplified signal, $X_{\mathrm{R}}[k]$ from Equation (6) into Equation (7) gives

$$\begin{aligned} Y_{\mathrm{D}}[k] &= \alpha[k]X[k]H_{r-1}[k]H_r[k] \\ &\quad + \alpha[k]X[k]\Phi_{\mathrm{Amp,S}}[k]\Phi_{\mathrm{Dop,SR}}[k]H_{r-1}[k]H_r[k] \\ &\quad + \alpha[k]W_{\mathrm{R}}[k]H_r[k] \\ &\quad + X[k]\Phi_{\mathrm{Amp,R}}[k]\Phi_{\mathrm{Dop,RD}}[k]H_r[k] + W_{\mathrm{D}}[k] \end{aligned} \tag{8}$$

We consider that the HPA distortions and Doppler shift effects are introduced at each hop the OFDM symbol traversed through [11,32]. The received signal power $E[|Y_{\mathrm{D}}[k]|^2]$ at the destination can be expressed as

$$\begin{aligned} E[|Y_{\mathrm{D}}|^2][k] = {} & \underbrace{|\alpha[k]|^2 P_{\mathrm{S}}|H_{r-1}[k]|^2|H_r[k]|^2}_{\text{desired signal}} \\ & + \underbrace{|\alpha[k]|^2 P_{\mathrm{S}}|H_{r-1}[k]|^2|H_r[k]|^2\sigma^2_{\Phi_{\mathrm{Amp,S}}}\sigma^2_{\Phi_{\mathrm{Dop,SR}}} + \sigma^2_{\Phi_{\mathrm{Amp,R}}}\sigma^2_{\Phi_{\mathrm{Dop,RD}}}P_{\mathrm{R}}|H_r[k]|^2}_{\text{ICI}} \\ & + \underbrace{|\alpha[k]|^2\sigma^2_{\mathrm{R}}|H_r[k]|^2 + \sigma^2_{\mathrm{D}}}_{\text{noise}} \end{aligned} \tag{9}$$

The SINR of the two hops can be defined as follows:

$$\begin{aligned} \gamma_{r-1}[k] &= \frac{P_{\mathrm{S}}|H_{r-1}[k]|^2}{\sigma^2_{\Phi_{\mathrm{Amp,S}}}\sigma^2_{\Phi_{\mathrm{Dop,SR}}} + \sigma^2_{\mathrm{R}}} \quad \text{and} \\ \gamma_r[k] &= \frac{P_{\mathrm{R}}|H_r[k]|^2}{\sigma^2_{\Phi_{\mathrm{Amp,R}}}\sigma^2_{\Phi_{\mathrm{Dop,RD}}} + \sigma^2_{\mathrm{D}}} \end{aligned} \tag{10}$$

Therefore, the end-to-end SINR, after defining the received signal power $E[|Y_{\mathrm{D}}[k]|^2]$ terms, the desired signal P_{des}, the distortion signal P_{ICI}, and the noise term P_{noi}, can be expressed as

$$\gamma[k] = \frac{P_{\mathrm{des}}}{P_{\mathrm{ICI}} + P_{\mathrm{noi}}} \tag{11}$$

After substituting the received signal terms P_{des}, P_{ICI}, and P_{noi} and considering the SINR in Equation (10) with some manipulations, Equation (11) can be simplified as

$$\gamma[k] = \frac{\dfrac{\gamma_{r-1}[k]\gamma_r[k]}{\left(\sigma^2_{\Phi_{\mathrm{Amp,S}}}\sigma^2_{\Phi_{\mathrm{Dop,SR}}}\gamma_{r-1}[k]+1\right)\left(\sigma^2_{\Phi_{\mathrm{Amp,R}}}\sigma^2_{\Phi_{\mathrm{Dop,RD}}}\gamma_r[k]+1\right)}}{\left(\dfrac{\gamma_{r-1}[k]}{\sigma^2_{\Phi_{\mathrm{Amp,S}}}\sigma^2_{\Phi_{\mathrm{Dop,SR}}}\gamma_{r-1}[k]+1}\right) + \left(\dfrac{\gamma_r[k]}{\sigma^2_{\Phi_{\mathrm{Amp,R}}}\sigma^2_{\Phi_{\mathrm{Dop,RD}}}\gamma_r[k]+1}\right) + 1} \tag{12}$$

The end-to-end SINR formula shows the HPA nonlinearity distortion and Doppler shift effects on the performance of the relay channels. Those effects are based on the Φ_{Amp} of the transmitters, MS or RS, and Φ_{Dop} of the relay channel.

3. BIT ERROR RATE PERFORMANCE ANALYSIS

In this section, we analyze the BER performance of the two-hop relaying OFDM system in the presence of HPA distortions and Doppler shift effects. A closed-form expression for the probability of error $P_{\mathrm{e}}(k)$ is derived and then used to calculate the SER as well as to measure the BER performance.

3.1. Signal-to-interference-plus-noise ratio expression

We concentrate on the uplink communication, where the source is considered to be a mobile entity with non-line-of-sight communication link with the RS; therefore, the MS–RS channel, $H_{r-1}[k]$, is modeled as a Rayleigh fading. Thus, the first-hop SINR for the kth subcarrier is considered to be an exponential random variable, which has the probability density function expressed as

$$f_{\gamma_{r-1}}[k](z) = \left(\frac{1}{\overline{\gamma}_{r-1}[k]}\right) \mathrm{e}^{\left(\frac{-z}{\overline{\gamma}_{r-1}[k]}\right)} \tag{13}$$

where the average SINR, $\overline{\gamma}_{r-1}[k]$, equals $E[\gamma_{r-1}[k]]$. The communication in the second hop is between two fixed entities (i.e., base station and relay station); therefore, the RS–BS channel, $H_r[k]$, can be considered as a static. Thus, $\gamma_r[k] = \overline{\gamma}_r[k]$ for all k. On the other hand, the frequency selectivity can be achieved by allowing different $\overline{\gamma}_r[k]$ for different k.

3.2. Probability of error expression analysis

The probability of error $P_{\mathrm{e}}(k)$ expression of the system over Rayleigh fading channels for M-ary QAM modulation scheme [33] is given by

$$P_{\mathrm{e}}(k) = \frac{1}{\sqrt{M}\log_2\sqrt{M}} \sum_{j=1}^{\log_2\sqrt{M}} \sum_{n=0}^{v_j} p_n^j \times \operatorname{erfc}\left(\sqrt{g_n \gamma[k]}\right) \tag{14}$$

where $v_j = (1-2^{-j})\sqrt{M}-1$, $p_n^j = (-1)^{\left\lfloor \frac{n2^{j-1}}{\sqrt{M}} \right\rfloor}[2^{j-1} - \lfloor (n2^{j-1}/\sqrt{M}) + (1/2) \rfloor]$, $g_n = [(2n+1)^2 3\log_2 M]/(2M-2)$, M is the constellation order, erfc is the complementary error function [34, Eq.(4A.6)], and $\lfloor\cdot\rfloor$ is the floor. The average probability of error $\overline{P}_{\mathrm{e}}(k)$ can be calculated by averaging $P_{\mathrm{e}}(k)$ in Equation (14) over the distribution of the fading channel as

$$\overline{P}_{\mathrm{e}}(k) = \int_0^{\infty} P_{\mathrm{e}}(k)\big|_{\gamma_{r-1}[k]=z}\, f_{\gamma_{r-1}[k](z)}\,\mathrm{d}z \tag{15}$$

Substituting Equation (13) into Equation (15) gives

$$\overline{P}_{\mathrm{e}}(k) = \int_0^{\infty} \frac{1}{\sqrt{M}\log_2\sqrt{M}} \sum_{j=1}^{\log_2\sqrt{M}} \sum_{n=0}^{v_j} p_n^j \times \operatorname{erfc}\left(\sqrt{g_n \gamma[k]}\right) f_{\gamma_{r-1}[k](z)}\,\mathrm{d}z \tag{16}$$

Substituting the probability density function for the SINR of the first hop, $f_{\gamma_{r-1}}[k](z)$, Equation (16) can be rewritten as

$$\overline{P}_{\mathrm{e}}(k) = \int_0^{\infty} \frac{1}{\sqrt{M}\log_2\sqrt{M}} \sum_{j=1}^{\log_2\sqrt{M}} \sum_{n=0}^{v_j} p_n^j \times \operatorname{erfc}\left(\sqrt{g_n \gamma[k]}\right) \left(\frac{1}{\overline{\gamma}_{r-1}[k]}\right) \mathrm{e}^{\left(\frac{-z}{\overline{\gamma}_{r-1}[k]}\right)} \mathrm{d}z \tag{17}$$

which can be rearranged as

$$\overline{P}_{\mathrm{e}}(k) = \frac{1}{\sqrt{M}\log_2\sqrt{M}} \sum_{j=1}^{\log_2\sqrt{M}} \sum_{n=0}^{v_j} p_n^j \times \int_0^{\infty} \operatorname{erfc}\left(\sqrt{g_n \gamma[k]}\right) \left(\frac{1}{\overline{\gamma}_{r-1}[k]}\right) \mathrm{e}^{\left(\frac{-z}{\overline{\gamma}_{r-1}[k]}\right)} \mathrm{d}z \tag{18}$$

The closed-form formula for $\overline{P}_{\mathrm{e}}(k)$ can be expressed as follows:

$$\overline{P}_{\mathrm{e}}(k) = \frac{1}{\sqrt{M}\log_2\sqrt{M}} \sum_{j=1}^{\log_2\sqrt{M}} \sum_{n=0}^{v_j} p_n^j \times \left(1 - \sqrt{\frac{g_n \overline{\gamma}_{r-1}[k]^{-1}}{1 + g_n \overline{\gamma}_{r-1}[k]^{-1}}}\right) \tag{19}$$

Using the obtained average probability of error, $\overline{P}_{\mathrm{e}}(k)$, the average SER of the OFDM symbol can be expressed as follows [35]:

$$\mathrm{SER}_{(\mathrm{OFDM})} = \frac{1}{N_{\mathrm{c}}} \sum_{k=0}^{N_{\mathrm{c}}-1} \overline{P}_{\mathrm{e}}(k) \tag{20}$$

Finally, the BER for subcarrier k in a two-hop relaying network can be measured using the obtained $\mathrm{SER}_{(\mathrm{OFDM})}$ as

$$\mathrm{BER} = \frac{\mathrm{SER}_{(\mathrm{OFDM})}}{\log_2(M)} \tag{21}$$

Section 4 presents numerical and simulation results for BER performance in the presence of HPA distortions and Doppler shift effects. The derivation of the BER performance is obtained for a WiMAX network model that has a maximum of two hops. The derivation can be extended to a general number of hops for any broadband network, for instance, the LTE.

4. SIMULATION RESULTS

In this section, we present simulation results to verify the BER performance analysis of OFDM systems in two-hop relaying network. From the end-to-end SINR expression in Equation (12), it is clear that HPA distortion and Doppler shift significantly affect the performance of the relaying channels. A special case is considered where the power amplifier is linear and the MS is stationary, $\Phi_{\mathrm{Amp}} = \pi/24$ and $\Phi_{\mathrm{Dop}} = 0$, respectively. In the simulation model, three signal constellations were used: 4-QAM, 16-QAM, and 64-QAM. The CP is added to each OFDM symbol to overcome intersymbol interference effects; the CP can take the values 25%, 12.5%, 6.25%, and 3.125% of the OFDM symbol length; however, in this work, only CP = 25% was used. The Doppler shift values used in the simulation were 0 and 100 Hz, whereas the HPA distortion values are $\pi/24$ and $\pi/6$, respectively. The transmission of sufficient OFDM symbols through $R = 2$ hops was simulated, and then the BER performance was measured.

Figures 4 and 5 present the BER performance of the single-hop ($R = 1$) and two-hop ($R = 2$) OFDM systems, respectively. The performance is measured using different QAM constellations (4-QAM, 16-QAM, and 64-QAM). For the illustration in these figures, the frequency shift is set to $f_{\mathrm{d}} = 0$, and the amplifier distortion is set to $\alpha = \pi/24$, which means that the MS is stationary and the HPA is applying the ideal linear characteristics. A perfect agreement was observed between the simulation and analytical results in both systems. This means that neither Doppler shift nor HPA distortion has an affect on the BER performance.

Figures 6 and 7 show the BER performance of the single-hop and two-hop OFDM systems, respectively. For the demonstration in Figures 6 and 7, we used the

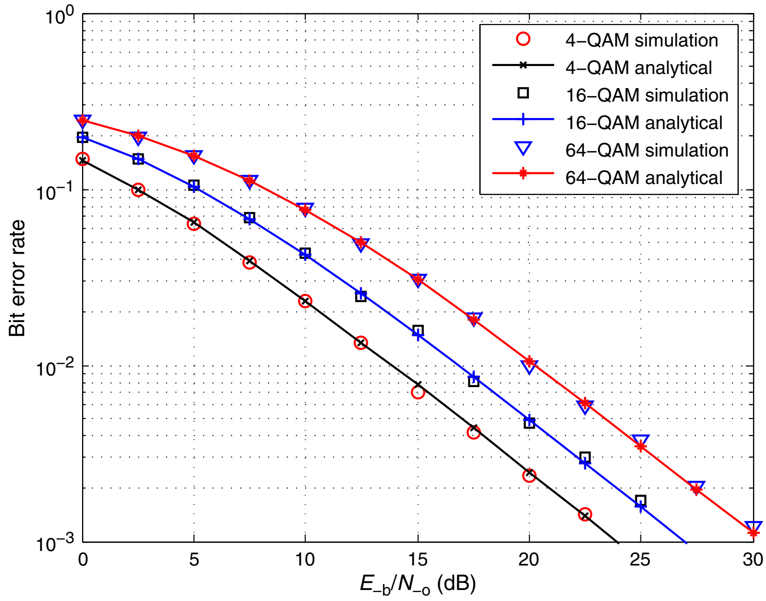

Figure 4. Bit error rate performance versus E_{-b}/N_{-0} of the orthogonal frequency-division multiplexing system over Rayleigh fading channel, single-hop, $R = 1$, $f_d = 0$ Hz, $\alpha = \pi/24$, $M =$ 4-QAM, 16-QAM, and 64-QAM.

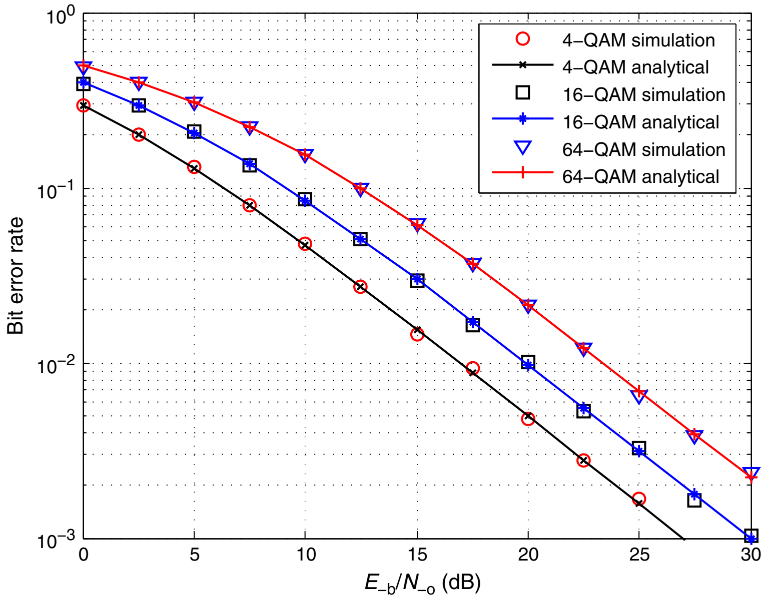

Figure 5. Bit error rate performance versus E_{-b}/N_{-0} of the orthogonal frequency-division multiplexing system over Rayleigh fading channel, two-hop, $R = 2$, $f_d = 0$ Hz, $\alpha = \pi/24$, $M =$ 4-QAM, 16-QAM, and 64-QAM.

following values: $f_d = 100$ Hz and $\alpha = \pi/6$. It can be observed from Figure 6 for the single-hop system that the error floor is presented because of the HPA distortion and Doppler shift impairments. In the case of the 16-QAM constellation, the error floor is higher than that in the case of the 4-QAM constellation order, whereas the highest error floor is measured for the 64-QAM constellation order. This is because more bits are transmitted that results to more bits being lost. Therefore, as the constellation order M increases, the ICI effect increases, resulting in a higher error floor.

Figure 7 illustrates the BER performance of the two-hop OFDM system. It can be observed that the error floor in the case of the 16-QAM constellation order and the two-hop relaying system is higher than that in the case of the single-hop system for the same constellation order. The

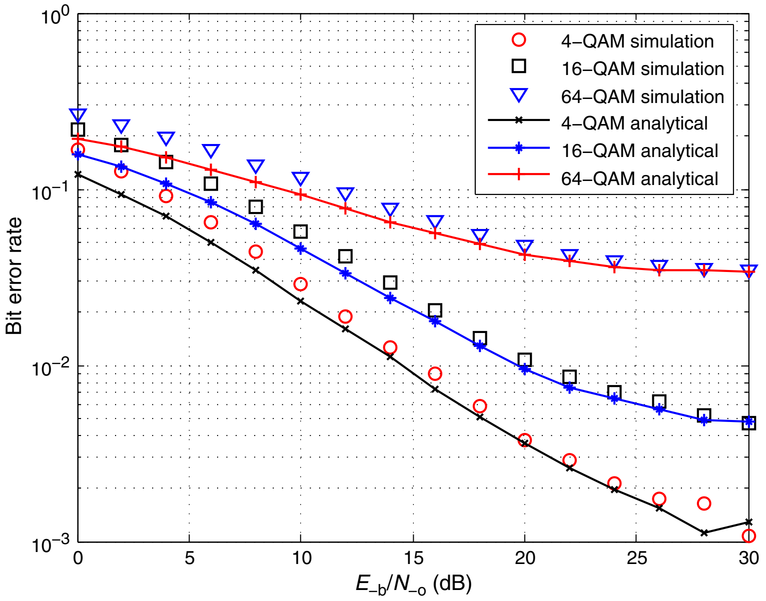

Figure 6. Bit error rate performance versus E_{-b}/N_{-0} of orthogonal frequency-division multiplexing system over Rayleigh fading channel, single-hop, $R = 1$, $f_d = 100$ Hz, $\alpha = \pi/6$, $M =$ 4-QAM, 16-QAM, and 64-QAM.

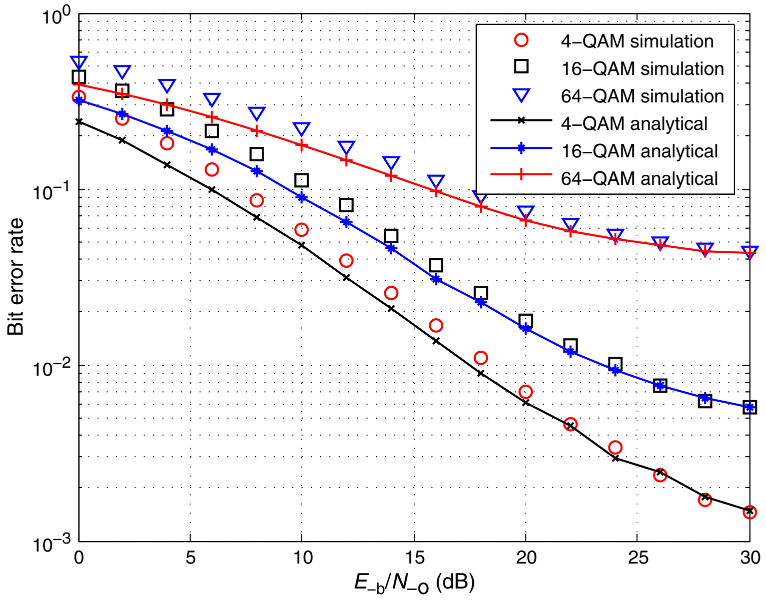

Figure 7. Bit error rate performance versus E_{-b}/N_{-0} of orthogonal frequency-division multiplexing system over Rayleigh fading channel, two-hop, $R = 2$, $f_d = 100$ Hz, $\alpha = \pi/6$, $M =$ 4-QAM, 16-QAM, and 64-QAM.

highest measured error floor is in the case of the two-hop relaying system and the 64-QAM constellation order because of the ICI component as a result of Doppler shift and HPA distortion as well as their cumulative effects on the two-hop relaying channels. Hence, the number of hops and the constellation order control the performance of the BER in the relaying system when channel impairments are present.

5. CONCLUSION

Analysis of the effects of amplifier nonlinearity and Doppler shift on the OFDM relaying system has been presented in this paper. The HPA shifts the transmitted signal before transmission over the channel, resulting in ICI. Also, the relative speed between the transmitter and the receiver shifts the transmitted frequency at the receiver side, also causing ICI. The resulting ICI due to the cumulative effects of the amplifier nonlinearity and Doppler shift per hop becomes very significant in a multihop relaying communication system. Analytical BER performance of the system was evaluated, and simulation results were presented to verify the analysis. A good match was observed between the analytical and simulation results. Based on some observations from the BER curves, the following conclusions can be drawn: (i) The OFDM system is sensitive to ICI, and this sensitivity is increased with the increase of the constellation order M. (ii) Amplifier distortion is one cause of the ICI, which depends on the amplifier characteristics. (iii) The Doppler shift is another cause of the ICI, which increases with the speed of the MS. (iv) The resulting ICI due to the cumulative effects of these impairments per hop becomes very significant on a multihop relaying communication system and severely degrades the BER performance of the system. The aforementioned issues should therefore be given due consideration to ensure acceptable BER performance in upcoming mobile multihop relaying system such as IEEE 802.16j and LTE. In addition, some more practical fading channel models such as the Nakagami and Rician models can be considered. The results could be quite different.

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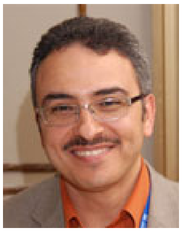

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