Bootstrap-based Quality Metric for Scarce Sensing Systems

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***Abstract*—This paper considers Mobile Crowd-Sensing (MCS) systems that suffer from scarce participant availability due to small sample sizes in each sensing cycle. With such small sample sizes, a sample in error would dramatically affect the MCS system performance. Therefore, we propose a novel quality of source metric targeted for small sample sizes through the non-parametric bootstrap, the trimmed mean, and the Median Absolute Deviation Trimming-based mean (MAD-mean). This statistic permits outlier detection, and therefore allows the estimation of quality under the stringent conditions of small sample sizes present in MCS independent sensing cycles. We introduce an algorithm that allows MCS administrators to control the accuracy of the metric, and therefore control the range of accepted values. Such control is achieved by means of introducing the MAD-mean, which deliberately widens the statistic's distribution, and therefore the perception of quality. In combination with the bootstrap, our metric allows quality estimation for samples as small as 8. We develop our robust quality of source metric algorithm, showing the impact of all the involved parameters; and we compare it to computer simulations to demonstrate its viability.**

***Index Terms*—mobile crowdsensing; internet of things; sensor networks; non-parametric bootstrap; small sample quality metric; source quality.**

I. Introduction

The Internet of Things (IoT) and mobile sensors' networks became a part of people's daily lives [1], and such integration led to the development of Mobile Crowd-Sensing (MCS) paradigm. MCS exploits the significant presence of sensors in the crowd, allowing system administrators to utilize the crowd, their sensor-loaded smartphones in particular, as an extended instrument to learn about their environment [1]. MCS became a hot topic in recent research due to its pervasiveness and the huge availability of potential sensing applications. It can be leveraged to sense the physical space, as well as the sociophysical behaviour of humans [2]. MCS can be classified into two main categories based on the nature of task execution: *opportunistic sensing*, in which passive execution takes place without any user intervention except his consent; and *participatory sensing*, that requires MCS participants to actively follow a described procedure to successfully complete sensing tasks [2].

An extensive portion of recent research contributions [3] [4] concentrate on Big Data, and propose techniques that assume data availability in abundance. However, MCS within the IoT-Ecosystem, should also operate on the assumption that Big Data is not always available. MCS data comes from a diverse variety of participant smartphones at different locations and different times. We believe that MCS solutions should tackle more realistic scenarios, where at the sensing layer the problem of data can be at *small data* scale, and not only Big Data. To that end, small-sample techniques are required for realistic IoT systems.

Small data techniques are complementary to Big Data techniques. The contrast between "Small" and "Big" lies in the fact that Big Data techniques seek to obtain a *global* picture, while the Small Data techniques seek to obtain a *local* picture. Such a distinction is important for applications targeting IoT and Smart Cities, as the global result stems from numerous local results. Also Small Data provides insight to administrators, policy makers, and researchers into the minute details of the system, which enables the design of more consistent policies and better understanding of the different scientific aspects involved [5].

Having small data sizes opens the door to alternative problems in realistic scenarios, where the administrators of MCS systems need to recruit members of the crowd to perform opportunistic or participatory sensing. Such recruitment needs to guarantee data quality while minimizing cost, where cost could be in terms of incentive payments [6], the provision of a service [2], or data consumed [7]. Ultimately, it is the number of participants that impacts the cost in an MCS system. However, in order to minimize cost and achieve an efficient system, the MCS administrator needs a mechanism, or a metric, that relates sensing quality and sample size, for each *spatio-temporal cell* [8].

Due to the variety and diversity of MCS participants, not all participants are of the same quality, as a result there is a heterogeneity present in the users' sensors [2]. This results in discrepancies in the accuracies and precision of the sensors that are trying to estimate the true value of the sensed quantity (for example, the normal distribution often comes up in sensed physical measurements). However, in an MCS system, the administrator is blind to the truth. This impairs the MCS administrator's ability to evaluate MCS participants. Such a problem would be minor in Big Data settings, but it is crucial in small data settings, as an incorrect measurement could drive the whole system. In such cases, the MCS administrators needs a statistical technique that would permit proper classification under the stringent conditions of a small sample size, and

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that is by means of proper detection of faulty sensors as *abnormalities* or *outlier samples* [9]. It is imperative that an MCS system be capable of automatic detection and isolation of such imperfections prior any analysis or inference about the true value, or even augmentation into Big Data. With that regard, this paper investigates a novel source quality metric for such imperfections present in MCS systems.

The approach we adopt relies intensively on the *non-parametric bootstrap* [1] to characterize the quality of a sample. Within our setup, the bootstrap is used to compute a sample distribution of a difference statistic, denoted θ_{MMTM}, that combines, in a controlled manner, the mean with robust centrality estimates: in particular the trimmed mean and the Median Absolute Deviation trimmed-mean (MAD-mean). This statistic provides a distribution that is similar to the bootlier plot. Our contribution is twofold: we provide a mathematical approach instead of the graphical approach in [10], and adapt it to commercial systems as the width of the distribution is deliberately widened by the presence of the MAD-mean term. The presence of the MAD-mean term is controlled by the MCS administrator via a parameter called sensitivity, that allows the administrator to impose tolerances on the outlier sensitivity of the statistic. This parameter is critical for commercial systems as it controls the relation between the desired quality and the allowed cost, that ultimately relates to the number of participants within an MCS system. We also propose an alternative quantification of the intuition captured in [10] by comparing that distribution to an ideal distribution, that of the dirac delta [11]. The introduced quality metric is based on the similarity between the statistic's distribution and that of the dirac delta.

The contributions of our paper lie in the development of the θ_{MMTM}-based metric, the statistical analysis of its mean and variance under the bootstrap, and the quality metric developed. The paper, in that sense, is organized as follows: Section II provides a review of the non-parametric bootstrap, the Bootlier; Section III describes the MAD-mean, the developed quality metric, its acquisition algorithm, and necessary mathematical rigor; Section IV illustrates the usage of the quality metric in a simulated scenario; and finally, Section V concludes with a discussion of potential improvements.

II. Review of Non-Parametric Bootstrap

A. The Non-Parametric Bootstrap

The non-parametric bootstrap [12] is a population-agnostic method, that allows the construction of sample distributions without prior assumptions about the population's distribution. Its numerical nature allows it to achieve its target by means of sampling with replacement a large number of times, B. The non-parametric bootstrap resamples the original sample $X = \{x_1, x_2, \ldots, x_N\}$, generating B resamples, $X_b = \{x_{b1}, x_{b2}, \ldots, x_{bN}\}$ where x_{bi} can appear more than once in X_b, where x_{bi} is a random variable that samples uniformly from X. To construct the sample distribution of a statistic θ, each X_b is employed to compute B θ_b statistics, gathered in a vector $\boldsymbol{\theta}^*$, whose histogram represents the sample distribution.

Despite its numerical complexity for a large B, the bootstrap is useful for crowdsensing applications where the scenario is sparse [1]. It is of particular usefulness for small-sample cases due to the fact that its fair resampling has a low probability, P, of selecting a homogeneous sample (i.e., all samples in X_b being exactly the same); which is obtained as:

$$P(X_b = \{x_i, x_i, \ldots, x_i\}) = [1 - (1 - 1/N)^N]^N \quad (1)$$

which is the probability of a binomial case. Inspecting the Eq. (1), out of B resamples, only 3.45% will be extremely biased for a sample of size $N = 8$.

The usefulness of the bootstrap in obtaining the quality of small-sample scenarios comes from the presence of outliers. Since resampling is uniform, the outliers presence is promoted to have a probability of $1/N$, like any other sample x_i in X. Further analysis of the binomial probability shows that the probability of an element's inclusion is:

$$P(x_i \in X_b) = 1 - (1 - 1/N)^N \quad (2)$$

which converges for a large N to 67%, i.e. each sample is present in 67% of the B X_b samples.

Furthermore, since B is a sufficiently large number, the properties of the central limit theorem are also applicable to the B θ_b statistics obtained. This property is of particular usefulness in the discussion of central measures of tendency (mean, median, mode,... etc), which causes the tendency of the bootstrap for the mean to follow a Gaussian distribution. However, due to the robustness of the MAD-mean, the resulting distribution is multi-modal because of the employment of the median in the MAD-mean's trimming process. It limits the set of *medians* to be selected to a discrete number of medians (for an odd N there are N possible medians, for an even N there are $N^2 - \sum_{i=0}^{N-1} i$ possible medians).

B. The Bootlier

The Bootlier, a graphical tool developed in [10], exploits the outlier promotion flaw in the bootstrap to detect outliers by computing the difference statistic between the mean $\bar{x}$ and the trimmed mean $\bar{x}_k$. The trimmed mean is a robust estimate of centrality with a breakdown point of $k\%$. The breakdown point is the percentage of outliers beyond which the statistic ceases to be robust. The mean is the least robust with a breakdown point of 0, and the median - and inherently the MAD - have a robust breakdown point of 0.5. In the Bootlier, the difference between the mean and the trimmed mean, $\bar{x} - \bar{x}_k$ is bootstrapped to construct a sample distribution. The histogram of the resulting sample distribution is called a bootlier plot, which shows the impact of outliers present in a sample. The authors in [10] investigated the bootlier's multimodality and smoothness, and developed the bootlier index as a measure to quantify its smoothness. However, their work heavily depends on human intuition for assessing the quality of a sample, and for detecting outliers, as well as does not provide a straight metric for the evaluation of a sample's *quality*.

We have analyzed the bootlier plot and found that its ideal reference is in fact the distribution $N(0, 0)$, which is best described as the dirac delta impulse [11], centered at 0. We have also found that the bootlier is a *superposition of leaking impulses*, which are very distinct when an outlier is present as it introduces another impulse at $x_o - \bar{x}_k$, where x_o is an outlier sample and $\bar{x}_k$ is the trimmed mean of the sample. The source of leakage is the variation within the original sample, which causes the resulting sample distribution, i.e. the bootlier, to be smooth. The bootlier statistic hides the resulting multimodalities which combine to form a seemingly normal distribution around 0, that should ideally - for an absolutely perfect sample - be an impulse.

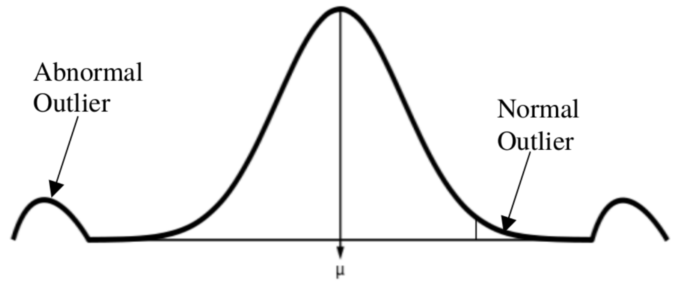

Fig. 1: Population with Abnormal Outliers (lower and upper).

III. Sample Quality Assessment

In the previous section, we have described how the non-parametric bootstrap is useful for getting the sample distributions of a statistic even from small sample sizes. We combine the MAD-mean with the bootstrap through the θ_{MMTM} statistic to develop a quality metric that accounts for both range and accuracy. For range, we employ the mean trimmed-mean difference, $\bar{x} - \bar{x}_k$, that is sensitive to outliers below the breakdown point of $k\%$. For accuracy, on the other hand, we employ the mean-MAD-mean difference, $\bar{x} - \bar{x}_{\text{MAD}}$, that is always robust about the breakdown point of 50%, which makes the metric more insensitive to the presence of outliers than the trimmed-mean $\bar{x}_k$. So, we introduce in this section a hybrid quality metric that utilizes this range-accuracy trade-off, along the MAD-mean and the θ_{MMTM} statistic.

A. MAD-based Trimmed Mean

For a sample $X = \{x_1, x_2, \ldots, x_N\}$, the dispersion of the population can be measured by means of the sample standard deviation. However, the sample standard deviation is a non-robust measure of dispersion which is easily affected by outliers, since it is a statistic that has a breakdown point of 0. The breakdown, as previously mentioned, is the point at which the statistic becomes blind and and no longer robust to outliers, as they become part of the population of the sample itself, which is an unrealistic scenario in practical systems. An alternative measure of dispersion is the Median Absolute Deviation (MAD), that has the robust breakdown point of the median, 0.5 [13]. The MAD is defined as the median of the absolute deviations from the sample median, stated as:

$$\text{MAD} = \text{median}\Big\{ \big| x_i - \overbrace{\text{median}(X)}^{\tilde{x}_M} \big| \Big\} \tag{3}$$

where x_i is the i^{th} sample in X, and the median is denoted $\tilde{x}_M$. It is a robust measure of dispersion that is based on *deviations from the median*, rather than deviations from the mean as is the case in the standard deviation. The MAD, however, is a *consistent estimator* of the standard deviation with:

$$\hat{\sigma}_{\text{MAD}} = \frac{1}{\Phi^{-1}(3/4)}\text{MAD} = \frac{1}{1.4826}\text{MAD}\Bigg|_{f(x)=N(\mu,\sigma)} \tag{4}$$

where $\hat{\sigma}_{\text{MAD}}$ is the MAD-based standard deviation estimator, $f(x)$ is the probability distribution followed by the population, $N(\mu, \sigma)$ represents the special case of the normal distribution centered around μ with a standard deviation of σ, Φ^{-1} is the quantile function $\big($which is the inverse of the cumulative distribution function $F(x)\big)$. The quantile function is evaluated at the probability of 75%, as the MAD is defined to cover the *median distance from the sample median* ($\tilde{x}_M$, which - for a symmetric distribution - covers 50% from the left to the right of the median $\tilde{x}_M$), and $\Phi^{-1} = 1.4826$ for any normal distribution $N(\mu, \sigma)$. We note that through the normal distribution, the general relation between the MAD and the standard deviation for any analytically defined distribution, $f(x)$, was derived [13].

The MAD inherits the robustness of the median, which makes it useful for the detection and removal of outliers. However, the definition of outliers is vague, and depends on the application in hand. For MCS systems, sensor measurements tend to follow the normal distribution, which involves *normal outliers* that rise from extremes of the distribution. However, there are *abnormal outliers* which lie far outside the three sigma range defined by the three sigma rule [14]. These abnormal outliers rather come from an *unexpected phenomenon* that is not modeled by the normal distribution, as illustrated in Fig. 1.

In an MCS system, these abnormal outliers need to be isolated as they represent a hindrance for the purpose of the MCS system, to estimate the truth at a specific time in a specific location. The MAD allows the removal of outlier samples present in X, producing a MAD-trimmed sample, denoted X_{MAD} [15], where *only values from within the range of the normal distribution are considered*, expressed as:

$$X_{\text{MAD}} = \{X : x_j \in [\tilde{x}_M \pm \underbrace{\lambda\text{MAD}}_{\delta\hat{\sigma}_{\text{MAD}}}]\} \tag{5}$$

where λ is how many MADs away from the median is the threshold beyond which samples are outliers. The product λMAD can be related to the consistent estimation of the standard deviation $\delta\hat{\sigma}_{\text{MAD}}$ to define it in terms of multiples (δ) of deviations from the mean ($\hat{\sigma}_{\text{MAD}}$).

Nevertheless, normal outliers are *expected*, unlike *abnormal outliers*, which lie farther beyond the three-sigma range. Thus, to ensure proper estimation of the true value, μ, samples beyond the 3-σ range could be considered as non-outliers, around $\lambda = 4$, and therefore only the values belonging to the interval $\tilde{x}_M \pm 4$MAD will be accepted. Therefore, the MAD-trimmed robust estimate of μ can be defined as:

$$\bar{x}_{\text{MAD}} = \frac{1}{N - N_o} \sum_{i=1}^{N-N_o} x_{\text{MAD}_i} \tag{6}$$

where N is the sample size, N_o is the number of outliers present (i.e., samples outside the $\tilde{x}_M \pm \lambda\text{MAD}$ range), and x_{MAD_i} is the i^{th} element in the MAD-trimmed sample, X_{MAD}.

Notice that the value of λ provides a degree of freedom for the system administrator to decide the range of measurements and the consideration of outliers. For the quality metric, λ takes values less than 2 for the purpose of maximizing robustness due to the small-sample sizes. The MAD-based outlier detection is a technique that is especially useful for small sample sizes [16], which makes it useful for the small-sample scenarios present in MCS systems' spatio-temporal cells. Algorithm 1 summarizes the procedure to obtain the MAD-mean.

Algorithm 1 Computing the MAD-mean.

Input: A sample: $X = \{x_1, x_2, ..., x_n\}$
Output: MAD-mean: $\bar{x}_{\text{MAD}}$
Initialize : λ
$\text{MAD}(X) = \text{median}(\text{median}(X) - X_i)$
for all $\boldsymbol{x_i}$ **do**
  if $x_i \notin [\texttt{median}(X) \pm \lambda\text{MAD}]$ **then**
    $X_o = \texttt{append}(x_i, X_o)$
  else
    $X_{\text{MAD}} = \texttt{append}(x_i, X_{\text{MAD}})$
  end if
end for
return $\bar{x}_{\text{MAD}} = \texttt{mean}(X_{\text{MAD}})$

B. MMTM Quality of Source Metric

The Bootlier, discussed in Section II, employs human intuition in its assessment, and is too sensitive to disregard normal outliers as abnormal outliers. Being overly sensitive to outliers always impacts the perception of the sample's quality, which is an important criterion in selecting the number of participants and thus impacts the overall system's cost. We have seen in the previous section how the MAD-mean is a robust centrality estimate for the true value, μ. We employ it to develop a quality metric that accounts for both range and accuracy. For range, one of the best solutions is the mean-trimmed mean difference, $\bar{x} - \bar{x}_k$, which exhibits acceptable sensitivity to outliers present within a sample with less than $k\%$ presence. For accuracy, the MAD-mean provides a suitable solution by considering the difference between the mean and the MAD-mean, $\bar{x} - \bar{x}_{\text{MAD}}$, that is always robust about the breakdown point of 50%, which makes it less sensitive to the presence of outliers than the trimmed mean $\bar{x}_k$. So, we introduce in this section a hybrid quality metric, that comes from the sample distribution of the *mean MAD-mean trimmed-mean difference* (MMTM) statistic, denoted θ_{MMTM}, that utilizes this trade-off between range and accuracy, which is defined as:

$$\theta_{\text{MMTM}} = \beta(\bar{x} - \bar{x}_k) + (1 - \beta)(\bar{x} - \bar{x}_{\text{MAD}}) \tag{7}$$

where $\bar{x}$ is the mean, $\bar{x}_k$ is the trimmed-mean, $\bar{x}_{\text{MAD}}$ is the MAD-trimmed mean, and β is the sensitivity parameter. The source quality metric has to relate to two important quantities: 1) the closeness of the θ_{MMTM}'s sample distribution to the ideal impulse best captured in terms of *location* and *spread*, and 2) the MCS administrator's desire to include or exclude normal outliers, defined by the sensitivity parameter β. Therefore, we define the quality metric Q_s based on the description of the θ_{MMTM} sample distribution as:

$$Q_s = \frac{1}{2}\left[\log_\gamma\left(\mu_{\text{MMTM}}^{-1}\right) + \log_\gamma\left(\sigma_{\text{MMTM}}^{-2}\right)\right] \tag{8}$$

where μ_{MMTM} and σ^2_{MMTM}, respectively, are the mean and variance of the θ_{MMTM} sample distribution, and $\log_\gamma$ is used to scale the values as later shown in the simulation section. The procedure to compute Q_s is summarized in Algorithm 2.

The feasibility region for our algorithm is wide and robust to samples whose outliers' proportion is less than the robust breakdown point of 0.5, beyond which the outliers are the majority of the sample itself.

Algorithm 2 Bootstrap Algorithm for Sample Quality Assessment.

Input: Readings from N sensors: $X = \{x_1, x_2, ..., x_n\}$
Output: Quality of Source: Q_s
Initialize : $B, \beta, k, \lambda, \gamma$
$\boldsymbol{\theta^*_{\text{MMTM}}} = \texttt{bootstrap}(X, B, \theta_{\text{MMTM}})$
$\mu_{\text{MMTM}} = \texttt{mean}(\boldsymbol{\theta^*_{\text{MMTM}}})$
$\mu_{\text{MMTM}} = \texttt{variance}(\boldsymbol{\theta^*_{\text{MMTM}}})$
return $Q_s = 0.5\left[\log_\gamma\left(\mu_{\text{MMTM}}^{-1}\right) + \log_\gamma\left(\sigma_{\text{MMTM}}^{-2}\right)\right]$

The mean of the resulting θ_{MMTM} distribution indicates where the distribution is located, and the variance indicates its spread. However, the best reference to compare with is the $N(0,0)$ distribution, the dirac-delta impulse. The Quality, Q_s, is defined as the average of the logs of the θ_{MMTM}'s mean and variance. The closer they are to zero, the higher the quality. This renders Q_s an absolute quality metric is it is free from any reference distributions or thresholds (excluding that of the impulse).

IV. Computer Simulation

In order to test our proposed algorithm we have considered a temperature evaluation scenario, where the samples are acquired from a distribution $N(24, 0.5)$, and the outliers were obtained from a distribution located at $+\bar{\Theta}\sigma$, $N(24+\bar{\Theta}\sigma, 0.3)$. Throughout this simulation, we use $\gamma = 10$, $k = 10$, $\lambda = 1.5$, and a number of bootstrap resamples $B = 1000$. Our algorithm has been evaluated under various scenarios, where we changed the sample sizes (N), and outlier deviations ($\bar{\Theta}$).

We first tackle the scenario of a variable sample size to show its impact on the obtained quality. That reflects the percentage of outliers and their impact, where the results are plotted in Figure 2. By increasing the sample size while N_o is fixed, then the percentage of outliers decreases, the quality will increase as shown in the figure. In order to realize the benefit of our proposed metric, another metric was used as a reference, $Q_s(\bar{x} - \tilde{x}_M)$ which considers only the difference between the mean $\bar{x}$ and the median $\tilde{x}_M$. The results show how MMTM always outperforms others. Moreover, Figure 2 illustrates the sample size (i.e., the number of sensors that should be recruited, with their corresponding cost) that is needed to achieve a predefined quality indicator.

We now tackle an alternative setup where we consider different kinds of sensors, and their impact on quality. It is represented through the Normalized Outlier Deviation, denoted $\bar{\Theta}$, that indicates how many standard deviations far from the true value μ do the outliers come from. Figure 3 illustrates the impact on the quality metric and how its largest value happens when it is at $\bar{\Theta} = 0$, i.e. no outliers are present, and decays as it goes farther from $\bar{\Theta} = 0$. We also considered another metric to help the reader realize the benefits of our proposed metric.

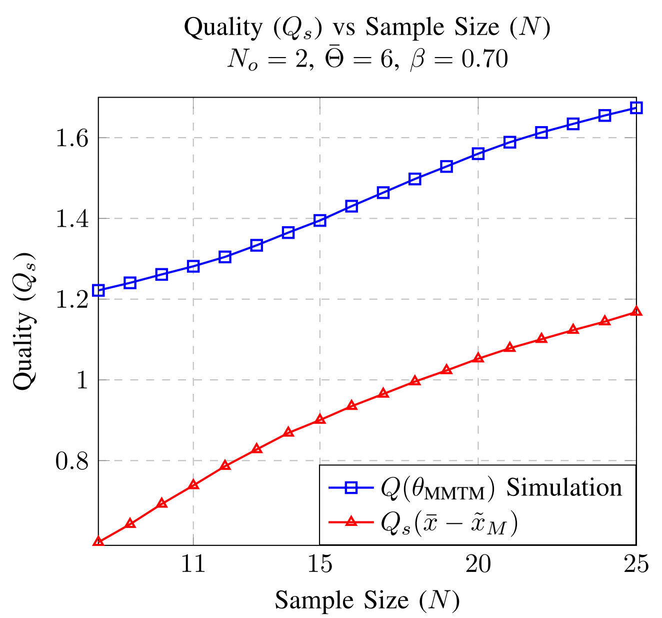

Fig. 2: MMTM quality metric for a variable sensitivity value

V. Conclusion

With the recent expansion in IoT and Big Data, small data provides a complement since systems, such as MCS systems, could also operate in conditions where data is scarce. This paper proposed the θ_{MMTM} statistic in combination with the non-parametric bootstrap to obtain a metric for quality for very small sample. The developed metric considers the administrator's desire in tolerating outliers.

The proposed metric employs the MAD-mean, the trimmed mean, and the non-parametric bootstrap for quality assessment

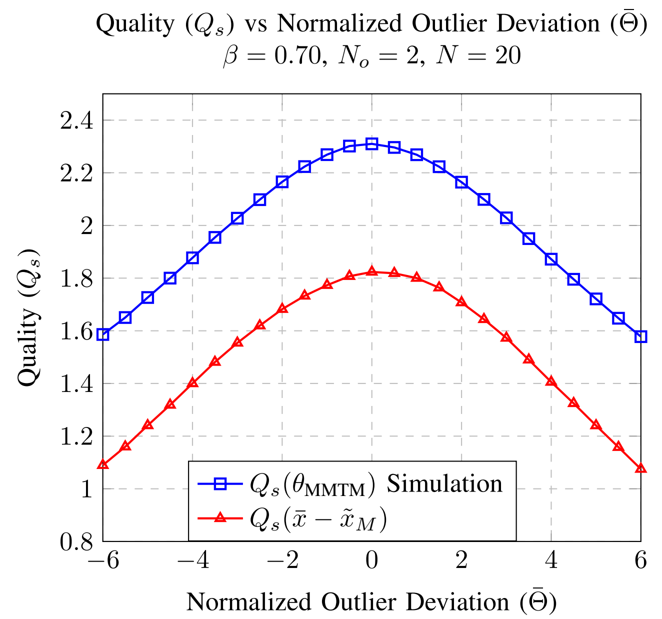

Fig. 3: MMTM quality metric for a variable normalized outlier deviation

for small samples in crowd-sensing applications. The developed metric provides the basis for a relation between quality and the sample size by analyzing the mean and the variance of θ_{MMTM} sample distribution. The significance of this quality metric lies in its attempt to quantify the adequacy of a sample for the purpose of the MCS system, and thus providing a basis for measuring the system's reliability in truth estimation. Our quality metric permits the MCS system and its affiliated stakeholder to be aware of their unawareness about the truth, while getting as close as possible to the truth. The quality metric developed is a simple tool to aid such insight.

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