

Capacity Analysis of Threshold-based Multiuser Scheduling in Broadband OFDMA Systems

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Abstract— This paper presents the capacity analysis of a threshold-based multiuser scheduling scheme in broadband OFDMA systems. Fourth-generation (4G) systems such as the WiMAX and LTE have adopted the band adaptive modulation and coding (band-AMC) subcarrier permutation mode for the grouping of the physical subcarriers into logical subchannels assigned to users by the base station (BS) schedulers. In the band-AMC system, the subcarrier grouped into the subchannels have correlated fading statistics. This makes an OFDMA scheduling scheme exploiting subchannel diversity attractive. In this work, we propose the use of energy threshold testing on each subchannel prior to the process of subchannel assignment by the scheduler. In the proposed scheme, users whose channel gains in the available subchannels equal or exceed a pre-determined energy threshold are scheduled for services at any time instant. This insures that only users who can maximize the capacity on the available subchannels are scheduled for data transmission, enhancing the BS data rate. We derive the capacity enhancement achievable using this scheduling scheme, and also present system-level simulations to corroborate the analysis. Both the analytical and simulation results indicate significant data rate enhancements using the proposed scheduling scheme.

Index Terms—Capacity analysis, threshold-based scheduling, broadband OFDMA, WiMAX networks, LTE networks.

I. INTRODUCTION

OFDMA scheduling in the 4G system has received significant attention lately as researchers and developers seek more efficient ways to handle the time-frequency resources in the 4G networks [1]-[2]. In the WiMAX networks, WiMAX OFDMA standard defines several subcarrier permutation options such as partial usage of subcarriers (PUSC), full usage of subcarriers (FUSC), and band adaptive modulation and coding (band-AMC), for the grouping of physical subcarriers into logical subchannels that form the basic unit of resource allocation by the base station (BS) schedulers [3]. In the PUSC permutation option, subchannels are formed by randomly mixing different subcarrier groups onto a subchannel, each group consisting of adjacent subcarriers in the frequency spectrum. In the FUSC permutation option, subchannels are formed by taking subcarriers directly from different portions of the spectrum in a completely pseudo-random manner, without any initial groupings. In the band-AMC permutation option, only subcarriers adjacent in the frequency spectrum are

included in a subchannel [3]. The statistics of the resulting subchannels, and consequently the performance of an OFDMA scheduling strategy rely heavily on the subcarrier permutation option used in the system. While the statistics of the subchannels in the PUSC and the FUSC systems are more difficult to characterize because of the random mixing process, the statistics of the subchannels in the band-AMC scheme can be characterized reasonably well, making system planning and performance predictions easier. For this reason, the 4G LTE standard has adopted the band-AMC as a subcarrier permutation option in the LTE networks.

In this work, we present the capacity analysis of a threshold-based multiuser scheduling scheme in broadband OFDMA systems for the band-AMC subcarrier permutation option. Threshold-based selection method was first proposed by Sulyman and Kousa in [4] for the diversity combining problem in a single-user transmission system, and has been widely studied in the literature [5]. In the context of multiuser scheduling in broadband OFDMA network, the author in [1] recently examines the use of a threshold-based multiuser scheduling, where a BS scheduler uses the energy threshold criterion [4],[5], to select the users to be scheduled for downlink transmission at any time instant. The advantage of this scheduling strategy is that at any time instant, only users whose channels are strong enough to sustain the network operator's target data rate are scheduled. This allows operators to maximize system throughput and is more useful for non-real-time traffic, which are delay tolerant. Scheduling of data transmissions to (non-active) users with temporarily weak channels can wait until their channel conditions improve. WiMAX BS schedulers typically simultaneously handles resource allocations for real-time traffic such as unsolicited grant service (UGS) and real-time polling service (rtPS), and non-real-time traffic such as non-real-time polling service (nrt-PS) and best effort (BE) services. Efficient utilization of the resources for non-real-time traffic as proposed in the threshold-based scheduling scheme frees up bandwidth resources for real-time traffic and optimizes overall resource utilizations in the network.

In reference [1] however, the author only defines a performance metric called the throughput gain, and analyze that metric as a measure of the enhancements possible using the threshold-based scheduling scheme. The actual data rate

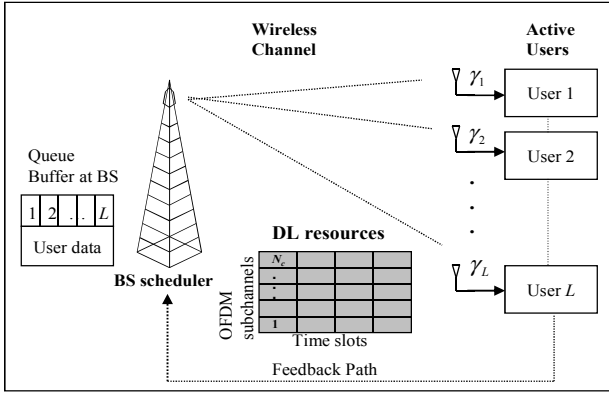


Fig. 1. Downlink scheduling in broadband OFDMA networks

enhancement experienced using threshold-based scheduling scheme is thus not yet analyzed. In this paper, we derive analytical expressions for quantifying the capacity enhancements provided by the proposed threshold-based multiuser scheduling scheme. The developed analysis is new in the literature, hence we complement the analysis with a system-level simulation. Both the analysis and simulation results indicate significant data rate (or throughput) enhancements using the proposed scheduling scheme.

II. SYSTEM MODEL AND ANALYSIS

A. System Model for threshold-based scheduling

Consider a downlink scheduler at a BS employing threshold-based multiuser scheduling scheme in broadband OFDMA network, as shown in Fig 1. On the i^{th} subchannel, the BS scheduler schedules n_i active users for downlink transmission, out of total of L users, whose SNR in that subchannel meet or exceed a predetermined energy threshold, γ_{th} . The data of these users then fill the (frequency, time) resources $(i, 1:n_i)$, $i = 1, 2, \dots, N_c$, in the next consecutive n_i OFDMA symbols transmitted by the BS. The number of users, n_i , satisfying the threshold requirement on the i^{th} subchannel at any time instant, is not fixed but variable in correspondence to user channel statistics on each subchannel. The specific realization of n_i could take any value from the set $\{1, 2, \dots, L\}$, at each scheduling period.

Let $\{\gamma_1, \gamma_2, \dots, \gamma_L\}$ denote the set of SNRs of the L users fed back to the BS on the i^{th} subchannel, and let $\{\gamma_{l:L}\}_{l=1}^L$ denote the order statistics obtained by arranging these SNRs in decreasing order of magnitude, (i.e., $\gamma_{1:L} \geq \gamma_{2:L} \geq \dots \geq \gamma_{n:L} \geq \gamma_{n+1:L} \geq \dots \geq \gamma_{L:L}$). As proposed in [4], define the threshold as

$$\gamma_{th} = \mu \cdot \gamma_{1:L} \quad (1)$$

where $0 \leq \mu \leq 1$. Then the BS scheduler conducts threshold test on the i^{th} subchannel and schedules the n_i users whose SNR rank in the set $\{\gamma_{1:L} \geq \gamma_{2:L} \geq \dots \geq \gamma_{n_i:L} \geq \gamma_{th}\}$.

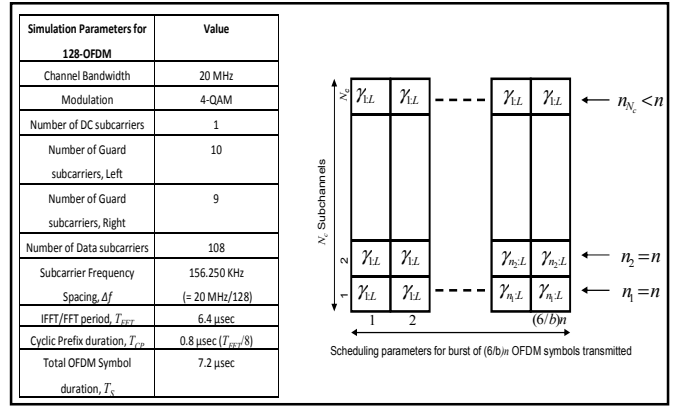


Fig. 2. Threshold-based multiuser scheduling in broadband OFDMA networks

Threshold-based multiuser scheduling as explained above is a generalized scheduling scheme that can be used to model several other broadband OFDMA scheduling schemes, depending on the threshold μ chosen at any time instant. For example for $\mu = 1$, the scheme reduces to opportunistic scheduling, and as μ is reduced, in the range $0 < \mu < 1$, more users are scheduled per channel use, and a whole range user fairness levels for proportional fairness (PF) can be modeled. The case $\mu = 0$ corresponds to the round-robin scheduling policy where all users with strong and weak channels are given equal transmission opportunities. Thus it is hoped that a threshold-based scheduler when implemented at the BS would provide a simpler, yet efficient, generalized scheduling policy that could combine the separate implementations of weighted round-robin (WRR), and PF, currently available in some WiMAX BS [6], into one simple algorithm where the threshold definition simply dictates the specific scheduling realization such as WRR, PF, etc., to be used at any time instant.

B. Capacity Enhancements for threshold-based multiuser scheduling with band-AMC subcarrier permutation mode

Broadband transmissions in 4G networks such as WiMAX system are typically arranged in bursts of variable lengths, specified in the downlink (DL) / uplink (UL) MAP messages at the beginning of each frame [3]. Assuming a burst of length $(6/b)n$ OFDM symbols, where b denotes the number of bins used in the band-AMC system. At any time instant, users feedback their SNR in each subchannel to the BS, using the assigned pilots in each bin of the band-AMC system, and a threshold-based multiuser scheduler conducts threshold test to select the n_i users whose SNRs, $\gamma_1, \gamma_2, \dots, \gamma_{n_i}$, are above threshold in the i^{th} subchannel and schedules them for service on that subchannel for $(6/b)n$ successive OFDM symbols transmitted in a burst, where $n = \max\{n_1, n_2, \dots, n_{N_c}\}$. For the case $n_i < n$ for a given subchannel, we assume that the BS fills the remaining time-frequency transmission slots opportunistically by allocating them to the user with the best SNR in that subchannel, as illustrated in Fig 2.

Thus without loss of generality, we assume $n_1 = n_2 = \dots = n_{N_c} = n$ in the analysis. The impact of this assumption is that the throughput performance evaluated in

the analysis is less than what would be obtained in practice using threshold-based scheduling in broadband OFDMA networks. Also for analytical convenience, we assume that $b = 1$ in the ensuing analysis. We later consider cases $b > 1$ in the simulations, as checks on the accuracies of the analytical results.

Since subchannels are partitioned in the band-AMC mode described above, we can consider the scheduling process in Fig 2 as a parallel N_c iid process, where n random variables are chosen out of L random variables in each process. Thus, it suffices that we focus on one subchannel in the analysis and later estimate the average capacity enhancements over N_c subchannels.

For DL transmissions in multiuser OFDMA system the BS capacity (or the normalized maximum BS data rate), which is an upper bound on the throughput possible per OFDM symbol, is given by:

$$r = \sum_{k=1}^{N_c} \log_2(1 + \gamma_k), \quad (2)$$

where γ_k denotes the SNR of the user being serviced on the k^{th} subchannel at any time instant. Focusing on one subchannel in the broadband OFDMA systems with band-AMC subcarrier permutation mode illustrated in Fig 2, and considering a burst of length $(6/b)n$, then we can express the instantaneous BS data rate per subchannel in the threshold-based scheduling system with band-AMC subcarrier permutation mode as

$$r = \frac{1}{(6/b)n} \sum_{l=1}^n (6/b) \log_2(1 + \gamma_{l:L}) \quad (3)$$

Thus, the average BS data rate per subchannel in the threshold-based multiuser scheduling scheme is given by

$$r = E_n \left\{ E_{\gamma_{1:L}, \dots, \gamma_{n:L}} \left[\frac{1}{n} \sum_{l=1}^n \log_2(1 + \gamma_{l:L}) \mid n \right] \right\} \quad (4)$$

Using the order statistics of the set of user SNR's $\{\gamma_{1:L}, \gamma_{2:L}, \dots, \gamma_{L:L}\}$, the average BS data rate per subchannel in the threshold-based multiuser scheduling scheme in Eq. (4) can be expressed as:

$$r = E_n \left\{ E_{\gamma_{1:L}, \gamma_{2:L}, \dots, \gamma_{n:L}} \left[\frac{1}{n} \sum_{l=1}^n \log_2(1 + \gamma_{l:L}) \mid n \right] \right\} = \sum_{n=1}^L \varphi(n) \cdot \Pr(\mathbf{n} = n) \quad (5)$$

where

$$\varphi(n) = E_{\gamma_{1:L}, \gamma_{2:L}, \dots, \gamma_{n:L}} \left[\frac{1}{n} \sum_{l=1}^n \log_2(1 + \gamma_{l:L}) \mid n \right] \quad (6)$$

and $\Pr(\mathbf{n} = n)$ denotes the probability that there are n users whose SNR equal or exceed $\gamma_{th} = \mu\gamma_{1:L}$.

To compute $\Pr(\mathbf{n} = n)$, we first observe that the event that the random variable $\mathbf{n} = n$ occurs when the following conditions are simultaneously satisfied [5]:

$$\begin{aligned} \mu\gamma_{1:L} \leq \gamma_{2:L} \leq \gamma_{1:L}, \mu\gamma_{1:L} \leq \gamma_{3:L} \leq \gamma_{2:L} \cdots \mu\gamma_{1:L} \leq \gamma_{n:L} \leq \gamma_{n-1:L} \\ \text{and } 0 \leq \gamma_{n+1:L} \leq \mu\gamma_{1:L}, 0 \leq \gamma_{n+2:L} \leq \gamma_{n+1:L}, \cdots \cdots, \\ 0 \leq \gamma_{L:L} \leq \gamma_{L-1:L} \end{aligned} \quad (7)$$

Using the *pdf* of $\{\gamma_{1:L}, \gamma_{2:L}, \dots, \gamma_{L:L}\}$, we compute $\Pr(\mathbf{n} = n)$ as

$$\begin{aligned} \Pr(\mathbf{n} = n) &= L! \int_0^\infty f_\gamma(\gamma_{1:L}) d\gamma_{1:L} \int_{\mu\gamma_{1:L}}^{\gamma_{2:L}} f_\gamma(\gamma_{2:L}) d\gamma_{2:L} \\ &\cdots \int_{\mu\gamma_{1:L}}^{\gamma_{n-1:L}} f_\gamma(\gamma_{n:L}) d\gamma_{n:L} \times \int_0^{\mu\gamma_{1:L}} f_\gamma(\gamma_{n+1:L}) d\gamma_{n+1:L} \\ &\int_0^{\gamma_{n+1:L}} f_\gamma(\gamma_{n+2:L}) d\gamma_{n+2:L} \cdots \int_0^{\gamma_{L-1:L}} f_\gamma(\gamma_{L:L}) d\gamma_{L:L} \\ &= n \binom{L}{n} \sum_{i=0}^{L-n} (-1)^i \binom{L-n}{i} \sum_{j=0}^{n-1} (-1)^j \binom{n-1}{j} \cdot \frac{1}{[1+j+\mu(n-1-j+i)]} \end{aligned} \quad (8)$$

Similarly, we compute $\varphi(n)$ as

$$\begin{aligned} \varphi(n) &= \frac{1}{n} \int_0^\infty \int_{\gamma_{n:L}}^\infty \cdots \int_{\gamma_{2:L}}^\infty \sum_{l=1}^n \log_2(1 + \gamma_{l:L}) n! \binom{L}{n} \\ &[1 - \exp(-a\gamma_{n:L})]^{L-n} \prod_{l=1}^n a \exp(-a\gamma_{l:L}) d\gamma_{1:L} \cdots d\gamma_{n:L} \end{aligned} \quad (9)$$

and the integral

$$\begin{aligned} I(n) &= \int_0^\infty \int_{\gamma_{n:L}}^\infty \cdots \int_{\gamma_{2:L}}^\infty \sum_{l=1}^n \log_2(1 + \gamma_{l:L}) n! \binom{L}{n} \\ &[1 - \exp(-a\gamma_{n:L})]^{L-n} \prod_{l=1}^n a \exp(-a\gamma_{l:L}) d\gamma_{1:L} \cdots d\gamma_{n:L} \end{aligned} \quad (10)$$

is solved using the transformation of the random variables $\{\gamma_{l:L}\}_{l=1}^n$ obtained by defining the spacings [7]

$$Y_1 = \gamma_{1:L} - \gamma_{2:L}, Y_2 = \gamma_{2:L} - \gamma_{3:L}, \dots, Y_n = \gamma_{n:L} \quad (11)$$

The final closed-form results for this integral is given by

$$\begin{aligned} I(n) &= (a)^n n! \binom{L}{n} \frac{1}{\ln 2} \left\{ \ln(n!) \prod_{l=1}^{n-1} \left(\frac{1}{al} \right) \cdot \int_0^\infty [1 - \exp(-ay_n)]^{L-n} \right. \\ &\exp[-any_n] dy_n + v_n \prod_{l=1}^{n-1} \left(\frac{1}{al} \right) \cdot \int_0^\infty \ln y_n [1 - \exp(-ay_n)]^{L-n} \\ &\exp[-any_n] dy_n + \sum_{k=1}^{n-1} v_k \prod_{l=1, l \neq k}^{n-1} \left(\frac{1}{al} \right) \cdot \left(\int_0^\infty \ln y_k \exp[-aky_k] dy_k \right) \\ &\left. \left(\int_0^\infty [1 - \exp(-ay_n)]^{L-n} \exp[-any_n] dy_n \right) \right\} \end{aligned} \quad (12)$$

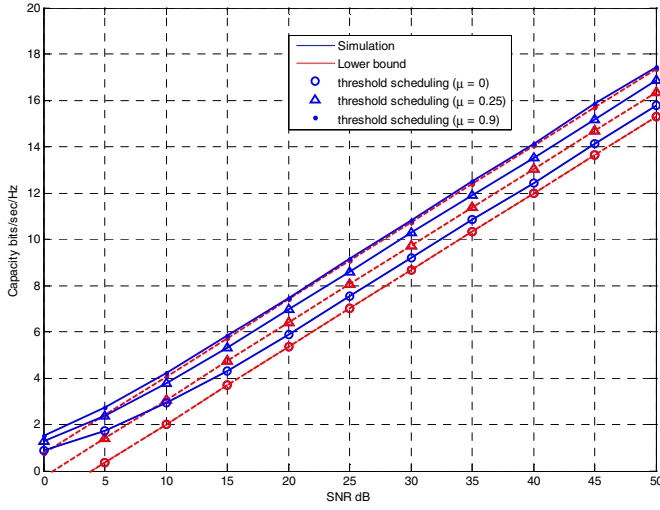


Fig. 3. Per subchannel capacity enhancement using threshold-based scheduling in broadband OFDMA (4 users per BS scheduler, band-AMC)

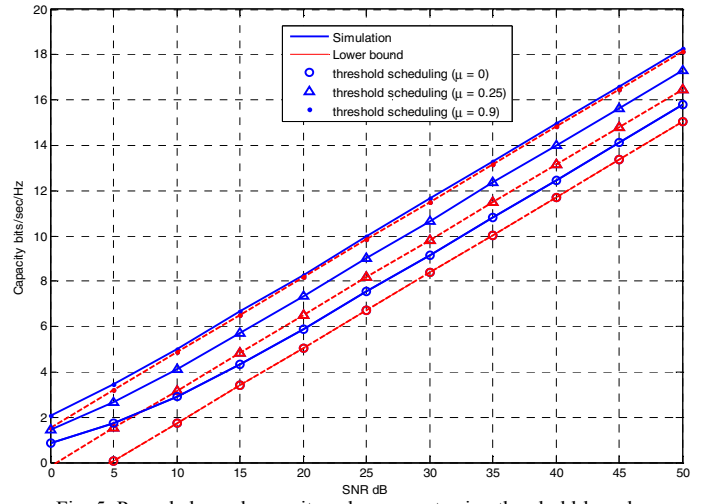


Fig. 5. Per subchannel capacity enhancement using threshold-based scheduling in broadband OFDMA (16 users per BS scheduler, band-AMC)

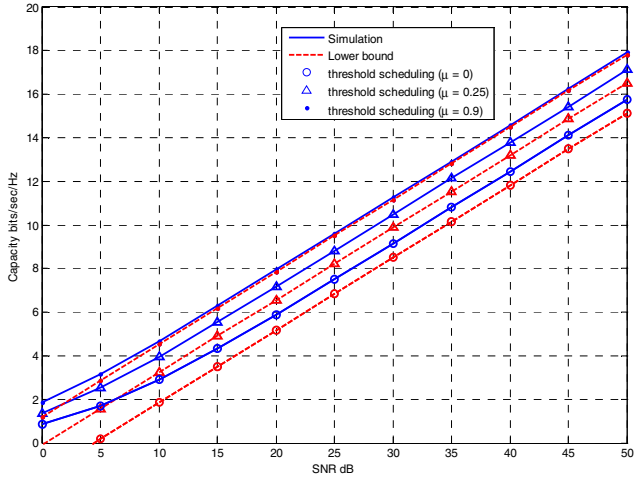


Fig. 4. Per subchannel capacity enhancement using threshold-based scheduling in broadband OFDMA (8 users per BS scheduler, band-AMC)

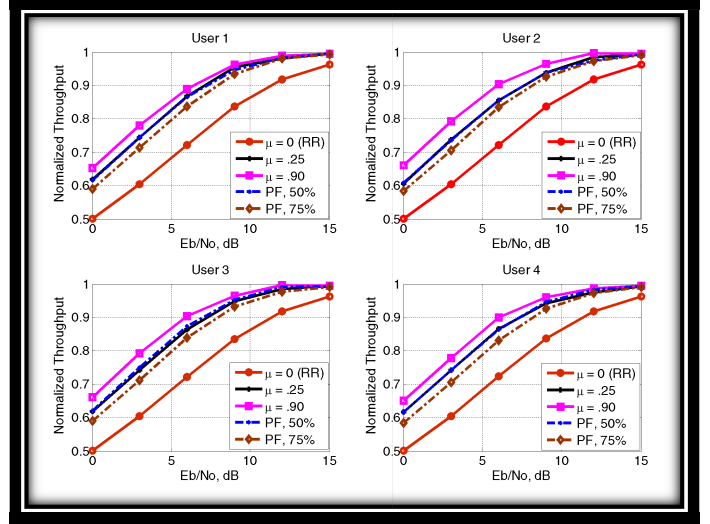


Fig. 6. Per user normalized throughput in threshold based scheduling for broadband OFDMA, band-AMC permutation mode

Using the following results from [8]

$$I_l = \int_0^{\infty} \ln y_l \exp[-aly_l] dy_l = \frac{-1}{al} [\mathbf{c} + \ln al]; \mathbf{c} \approx 0.5772.$$

$$I_{Ln} = \int_0^{\infty} [1 - \exp(-ay_n)]^{L-n} \exp[-any_n] dy_n$$

$$= \frac{1}{a} B[n, (L-n+1)]; B = \int_0^1 t^{x-1} (1-t)^{y-1} dt \quad (13)$$

we arrive at the final closed-form results for $I(n)$, after some algebra as

$$I(n) \geq \left(\frac{1}{\bar{\gamma}}\right)^n n! \binom{L}{n} \frac{1}{\ln 2} \left\{ \ln(n!) \prod_{l=1}^{n-1} \left(\frac{\bar{\gamma}}{l}\right) \cdot \bar{\gamma} B[n, (L-n+1)] \right.$$

$$\left. + v_n \prod_{l=1}^{n-1} \left(\frac{\bar{\gamma}}{l}\right) \cdot \sum_{q=0}^{L-n} \binom{L-n}{q} (-1)^q \left(\frac{-\bar{\gamma}}{(q+n)} \left[\mathbf{c} + \ln\left(\frac{1}{\bar{\gamma}}(q+n)\right)\right]\right) \right\}$$

$$+ \sum_{k=1}^{n-1} v_k \prod_{l=1, l \neq k}^{n-1} \left(\frac{\bar{\gamma}}{l}\right) \cdot \left(\frac{-\bar{\gamma}}{k} \left[\mathbf{c} + \ln \frac{k}{\bar{\gamma}}\right]\right) \left(\bar{\gamma} B[n, (L-n+1)]\right) \left. \right\} \quad (14)$$

Substituting Eq (14) in Eq (9), and using Eqs. (5) and (8), we obtain an expression for the per subchannel data rate performance of threshold-based scheduling scheme as

$$r \geq \sum_{n=1}^L \left\{ (n-1)! \left(\frac{1}{\bar{\gamma}}\right)^n \binom{L}{n} \frac{1}{\ln 2} \left[\ln(n!) \prod_{l=1}^{n-1} \left(\frac{\bar{\gamma}}{l}\right) \cdot \bar{\gamma} B[n, (L-n+1)] \right. \right.$$

$$\left. + v_n \prod_{l=1}^{n-1} \left(\frac{\bar{\gamma}}{l}\right) \cdot \sum_{q=0}^{L-n} \binom{L-n}{q} (-1)^q \frac{-\bar{\gamma}}{(q+n)} \left[\mathbf{c} + \ln\left(\frac{1}{\bar{\gamma}}(q+n)\right)\right] \right.$$

$$\left. + \sum_{k=1}^{n-1} v_k \prod_{l=1, l \neq k}^{n-1} \left(\frac{\bar{\gamma}}{l}\right) \cdot \left(\frac{-\bar{\gamma}}{k} \left[\mathbf{c} + \ln \frac{k}{\bar{\gamma}}\right]\right) \left(\bar{\gamma} B[n, (L-n+1)]\right) \right\} \cdot \Pr(n) \quad (15)$$

where $\Pr(n)$ is given by Eq. (8). Finally, the total estimate of the BS maximum data rate over all subchannels can be obtained for the threshold-based scheduling as $r_{threshold} = N_c r$.

III. SIMULATION RESULTS

A. Numerical Simulations

In Figs. 3-5, we plot the (lower-bound) analytical results for the per subchannel capacity performance of threshold-based multiuser scheduling in broadband OFDMA, using Eqs. (15) and (8). Counterpart results from numerical simulations of Eq. (4) are also presented in these figures for reference. It is observed from these figures that the lower-bound analytical expressions derived are very close to the actual numerical simulation of the BS capacity with only 5% gap maximum, in high SNR, for the case $\mu = 0$ which has the widest gap between the analysis and simulations. All other cases of μ have even much narrower gaps between the analysis and simulation. Thus the derived expressions are very useful for estimating the capacity enhancements of the proposed scheduling scheme.

From both the analytical and simulation results in Figs 3-5, it is observed that threshold-based scheduling scheme with threshold level as low as $\mu = 0.25$ enhance the capacity over the regular scheduling scheme (case $\mu = 0$) by over 1 bit/sec/Hz per subchannel per sector, which can be very significant in a sectorized cellular deployment involving large number of OFDM subchannels (e.g 1024-OFDMA with 16 subchannels). Higher enhancement is even achievable at higher threshold levels, albeit at the expense of user fairness. This observation is consistent for all cases depicted in Fig 3-5 (4-user, 8-user, and 16-user systems). Also comparing the results in Figs 3-5, it is observed that the capacity enhancements obtained using threshold-based scheduling scheme increases as more users are serviced per BS scheduler, taking advantage of the randomness of the user channel statistics when band-AMC subcarrier permutation mode is employed.

B. System-level Simulations

For our system-level simulation, we used the band AMC permutation mode for the OFDM subcarrier permutation option, and we used three consecutive bins over two consecutive symbols. This allows us to assess if the simulation results would report any deviation from the trends reported in the analytical results, for the cases when two or more pilots are assigned per subchannel since our analysis only model the case when one pilot (bin) is used per subchannel. Also in the simulation, we used the 128-FFT implementation defined in the WiMAX standard, with the parameters given in Fig 2, for a four user system. Fig 6 presents the per user throughput (or actual data rate) enhancements of the proposed scheduling scheme for different threshold levels μ . From this figure, we observe that in general, the trends observed in the analysis are also confirmed using system-level simulations. As a reference, we also include in this figure the performance of two popular WiMAX scheduling schemes, the round-robin (RR) and PF schemes. Round-robin is exactly the same as the proposed

threshold-based scheduling scheme for $\mu = 0$, and various levels of PF can be modeled for different values of μ . Also, and interestingly, we observe from the simulations that PF for 50% user fairness has exactly the same throughput enhancement as the threshold-based scheduling for $\mu = 0.25$. This value of μ thus represents a heuristic optimum that achieves good balance between user fairness and data rate enhancements in the threshold-based scheduling scheme.

C. *Cases of users with persistently weak channels*: For users with persistently weak channels, such users may not be scheduled for services using the proposed threshold-based scheduling algorithm for the case $\mu > 0$. The BS scheduler can have a policy to interrupt the proposed algorithm momentarily after every certain number of frames, by setting $\mu = 0$. This would lower the over-all BS data rate enhancements reported for the proposed scheme, but it helps ensure that data services for users with persistent weak channels can still continue.

IV. CONCLUSIONS

This paper proposes a threshold-based multiuser scheduling scheme for downlink transmission in broadband OFDMA systems. We quantify analytically the capacity enhancements, per subchannel, provided by the proposed scheme and also present simulation results to verify the analysis. Both the analyses and simulations indicate significant enhancements in BS data rate when large numbers of users are serviced per BS scheduler. It is hoped that a threshold-based scheduler when implemented at the BS would provide a simpler, yet efficient, generalized scheduling policy that could combine the separate implementations of WRR, and PF, currently available in some WiMAX BS [6], into one simple algorithm where the threshold definition simply dictates the specific scheduling realization such as WRR, PF, etc., to be used at any time instant.

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