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Data relaying with constraints in hierarchical sensor networks

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Abstract In this paper we present and discuss several optimisation problems that arise in the management of data flow in wireless sensor networks (WSNets). We consider a hierarchical architecture for WSNets composed of sensors, relays, and relay gateways. Sensors send data they generate at a known average bit rate to relays in one hop. The relay nodes use a multi-hop mechanism to reach a set of assigned gateways which then forward the data directly to the base station. We are concerned with finding an assignment of relay gateways to relays so that certain constraints are satisfied. We define a unified model in which constraints such as lifetime, data delay, and data flow splitting are formulated in terms of four optimisation problem in graphs.

Keywords Constraints · Hierarchical sensor networks · Wireless sensor networks · Data relaying

1. Introduction

Wireless sensor network (WSNets) are systems comprised of small independent devices called sensor nodes that monitor certain properties of the environment and use wireless signals to communicate their measurements (Akyildiz et al., 2002; Estrin et al., 2001; Lewis, 2004). In this paper we examine a specific hierarchical architecture of WSNets based on three layers of nodes. An important characteristic of our research is to consider the network topology given. This is suitable for applications where the placement of sensors cannot be controlled or as a second phase following node deployment in which data relaying decisions need to be taken. Our work provides, for the first time in this context, a theoretical foundation for the study of problems arising in the management of data flow in hierarchical wireless sensor networks. We are motivated by the need to specify these problems in a rigorous mathematical framework, specifically as optimisation problems. The contribution of this paper is therefore primarily to identify and articulate the questions to be addressed. As such,

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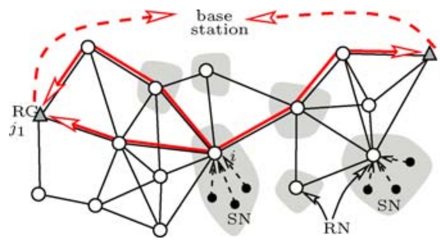


Fig. 1 An illustration of the 3-tier architecture. The SN are represented as gray regions in the plane; some RN collect data from SN and also relay data on behalf of other RN

our formulation leaves open the choice of those particular algorithms to be used for attacking these problems. It is hoped that the present effort will inspire research in this direction and offer a unified context for the comparison of different approaches.

In the following sections we characterise first the architecture of the WSNet we work with and we briefly review some of the similar structures present in the literature. We introduce two types of constraints that affect how data are relayed. Based on whether these constraints are enforced or not, we discuss in Section 2 four types of optimisation problems that model a relaying strategy in our system. In Section 3 we conclude with a discussion of open problems and future directions for research.

ARCHITECTURE

We propose a heterogeneous system comprised of three types of nodes—sensors, relays, and relay gateways (see fig. 1). The functions assumed by each layer in the architecture are:

1. Sensors (SN)—gather and send data values to one (or possibly more) relay node(s).
2. Relay nodes (RN)—receive data values from sensors and/or other relay nodes and forward them to relay nodes or relay gateways.
3. Relay gateways (RG)—receive data values from RN and send them directly (in one hop) to the base station.

We believe that such an architecture is more cost effective for applications where

(a) the points of interest monitored are spread over large distances,
(b) a large number of different measurements are collected, and
(c) the bit-rate of data processed is high.

In such cases, it makes more sense to use a large number of simple and cheap sensors that assume only the role of sensing, and two other classes of nodes—the relays and relay gateways—for data dissemination.

The problem proposed here is to select, for each RN, one (or possibly more) RG that should receive the flow of data from the RN. Different constraints such as residual energy or delay, can influence the strategy to allocate relay gateways to relay nodes. We model them by assigning capacity functions to nodes and hop constraints on relaying paths. In assigning RG to RN we also compute, as a by-product of our methods, one (or more) relaying paths to carry the data from the relay node to the relay gateway.

Research on hierarchical WSNet has chosen mostly two directions, placement of relay nodes to ensure connectivity and routing. We do not consider placement here, as our topology is given. The key difference between our approach and routing in wireless networks, whether they are sensor systems or ad-hoc, is that we compute an assignment of gateways to relay nodes and in doing so we also find relaying paths. In contrast, traditional routing protocols enforce first a clustering (an assignment of nodes to cluster heads) of the nodes and then compute a routing based on given source and destination node(s). It is beyond the scope of this paper to review the literature on routing and sensor node placement. We prefer to highlight some of the contributions concerning placement (Tang, 2005; Mhatre, 2005; Wang, 2005) and clustering for routing (Basagni, 1999; Banerjee and Khuller, 2001; Heinzelman et al., 2000; Younis and Fahmy, 2003; Pottie, 1998; Dyck and Miller, 2001).

2. Problem formulations

We represent the wireless sensor network as a directed graph $D = (V, A)$ where the set of vertices V is comprised of the set of relay nodes R and relay gateways G, $V = R \cup G$. An edge (i, j) exists between nodes i and j if i can send data directly to j. Every sensor node SN is assigned to one relay node and the data collected by this sensor node is forwarded to the the relay node. The average amount of data in a unit of time that is passed to a relay node i from all its sensor nodes is considered known and is denoted $\beta(i)$. In fact, $\beta : V \to \mathbb{Z}_+$ is a function defined over the set of vertices such that $\beta(k) = 0$ for all relay gateways k. We call $\beta(i)$ the *demand* of node i.

We represent the flow of data, i.e. the amount of data forwarded by a node i to a node j through a link, by a non-negative integer associated with the directed arc (i, j). These integers define functions over the arcs of the graph called *flow functions*. We also define a capacity function $\gamma : V \to \mathbb{Z}_+$ for each node in the graph. The capacity function is calculated for each node so that to insure its survivability given the global lifetime constraint of the system and represents the initial amount of energy for the corresponding node. The calculation must take in account the residual energy of the node and the fixed transmission range of the node to estimate the quantity of data that the node can relay. This quantity, divided by the value of lifetime that is guaranteed, gives the value of the capacity function that can be used as an upper-bound on the flow reaching the node.

We define another function $c : A \to \mathbb{Z}_+$ called the cost function. This function might represent the amount of energy consumed for transmitting a unit of data flow through each link. Function c defines the objective function to be minimized for each of the optimisation problems considered here. In other words, we find allocations of relay gateways to relay nodes subject to a combination of constraints and using a minimum amount of energy in communication.

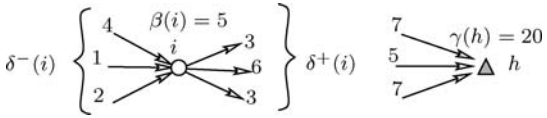
Given node i, we denote the set of all arcs entering i by $\delta^-(i)$, and all arcs leaving i by $\delta^+(i)$ (see fig. 2),

$$\delta^-(i) = \{(j, i) | (j, i) \in A\}, \tag{1}$$

$$\delta^+(i) = \{(i, j) | (i, j) \in A\}. \tag{2}$$

If S represents a subset of the arcs of the graph and $x : A \to \mathbb{N}$ a function over the set of arcs, we use $x(S)$ to represent the sum of values of function x over the elements of S, $x(S) = \sum_{a \in S} x(a)$.

Fig. 2 Notations used in the formulation of the routing problem and an example



Application specific constraints

We identified two types of constraints that influence the allocation of relay gateways to relay nodes. The first controls the delay incurred by relaying data from RN a to an assigned RG b. The base station receives data originating at a with a delay that is proportional to the number of intermediary nodes on the path from a to b. In delay sensitive environments it is important to guarantee that the delay is not greater than a certain limit. In this case, we can only return relaying paths that have a length smaller or equal than a given constant θ. Length is measured in the number of vertices on the path excluding the source.

The second constraint controls the number of different paths that relay data from a single RN. There are practical motivations for defining such a constraint. If relaying uses several paths and potentially more than one relay gateway, more effort needs to be invested in recombining the data at either the base station or at the relay gateway. Sometimes, it is preferred that such complications be avoided altogether, especially if data aggregation operations are performed by relay gateways Krishnamachari et al. (2002).

We thus define four different optimisation problems that are of interest for the management of data flow in hierarchical WSNet.

- Problem MP: *unconstrained Multi-Path relaying*. Here, data are relayed to relay gateways through one or more paths of arbitrary length in graph D. The flow of data may be split among different paths, so that node capacity constraints are satisfied.
- Problem MPD: *Multi-Path relaying with Delay constraints*. Data are relayed through one or more paths in D and the paths are not longer than a certain fixed upper bound. Data are split among the paths used.
- Problem SP: *unconstrained Single-Path relaying*. Data are relayed to exactly one relay gateway and the number of intermediary nodes data travels through is unbounded. Data originating at a relay node r must be relayed through a single path to one relay gateway.
- Problem SPD: *Single-Path relaying with Delay constraints*. Data are relayed through exactly one path as in problem SP but the relaying path have a length bounded by a constant.

2.1. Unconstrained multi-path relaying

Problem MP is the one with the fewest constraints imposed among all four problems. We are only required to make sure that all the data packets collected by relay nodes reach one or more relay gateways, and we make the assignment of RG to RN so that the minimum amount of energy is spent. A single flow function $x : A \rightarrow \mathbb{N}$ can model the routing decisions of every RN in the system. Given x, every relay node i will decide on the number of data packets sent on each direct link $a \in A$ that is incident on i and this is enough to insure the survivability of the network. In fig. 2 for example, node i expects to receive 4, 1, and 2 units of data through each incoming link, 5 units of data from the SN nodes in its vicinity, and it will simply send 3, 6, and 3 packets through each of the outgoing links. It does not matter what kind of data is sent through each link. We will see that for the remaining three problems, using a single flow function is not enough.

We define Problem MP as

$$\min_x \sum_{a \in A} x(a)\, c(a)$$

$$\text{subject to:} \quad x(\delta^+(i)) - x(\delta^-(i)) = \beta(i) \quad \forall i \in R \qquad (3)$$

$$x(\delta^-(i)) + \beta(i) \leq \gamma(i) \quad \forall i \in R \cup C.$$

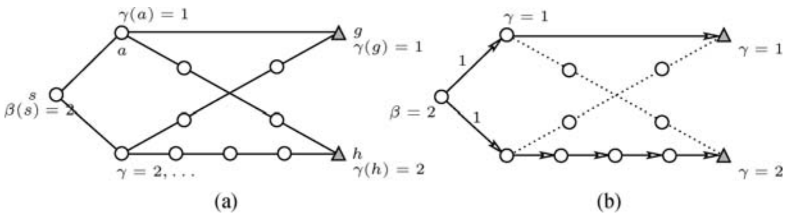


Fig. 3 (a) The WSNet graph for the examples in (b); g and h are relay gateways, the remaining vertices are relays; $\beta(s) = 2$ is the only non-zero value of function β; all relay nodes have capacity 2 except for vertex a. (b) An optimal solution for the unrestricted multi-path relaying problem for the graph in (a). The cost is 7

The first equation simply states that any relay node i must transmit all the data it receives from its neighbours and in addition, it must send the amount $\beta(i)$ of data coming from its allocated SN. In this way we are only interested in computing functions x that could represent flow of data in the network. The second inequality represents the capacity constraint. For relay gateways $\beta(i) = 0$. Equation (3) contains the classic flow conservation law found in network flow problems (Schrijver, 2003) (vol. A, Chapters 10–15) together with a generalisation of the capacity constraints usually associated with the graph edges in these problems. Figure 2 illustrates a function x that satisfies the constraints.

For a complete example, let us consider the WSNet graph in fig. 3(a) where all arcs are bidirectional and the cost function on arcs is constant $c = 1$. The relay nodes are drawn as circles and the relay gateways as triangles. For simplicity, we chose vertex s as the only relay node receiving data from sensors. All other relay nodes just forward information. For the input shown in fig. 3(a), an optimal solution for problem MP is depicted in fig. 3(b), where the arcs with non-zero flow are drawn with solid lines. The best way in terms of cost function c to send data from s to one or both gateways g and h in this graph is through the two paths on the top and the bottom. The top path is the shortest and has a cost of 2 but the vertices on it can only receive unit flow because of their capacity. Also, gateway g is saturated and thus the only path along which the remaining data can be transmitted is the bottom path with a cost of 5. The total cost of the solution is 7.

2.2. Multi-path relaying with delay constraints

When the time interval from the moment data are created until they reach the base station needs to be bounded above by a constant, we need additional information to allow us identify the origin of data relayed from the source to the destination. Therefore, the approach from Section 2.1 is no longer suitable. Our goal is to limit the relaying path length to at most a given constant θ.

We choose two different ways to formulate the path length constraint, one using flow functions as for the previous problem, and another using an enumeration of the set of feasible paths in the graph. Although the former formulation seems impractical because of the sheer number of such feasible paths in a non-trivial graph, it is possible to implement it in practice. In fact, the feasible path formulation is preferable to the one using flow functions for systems where memory requirements are critical. Similar models have been previously used in the context of wired network design problems (Gouveia et al., 2003).

Flow functions model: Consider a family of flow functions $x^r : A \to \mathbb{N}$ for all relay nodes r. Function x^r represents the flow of data collected by relay node r from its sensor nodes.

Let θ be the upper bound on the number of nodes that data are relayed through including the target relay gateway. To model the number of intermediary nodes that the flow originating at some RN r traverses, we assign an index to flow x^r that represents the hop count of the data represented by that flow function. Whenever a vertex relays the flow of data further, this index is incremented. If we denote this index by h, then our problem becomes to compute a family of functions $x_h^r : A \to \mathbb{N}$, for all $r \in R$ and $0 \le h \le \theta$, given that certain constraints are satisfied. These constraints are:

- Flow conservation relations similar to those in (3),

$$x_0^r(\delta^+(r)) = \beta(r) \quad \forall r \in R, \tag{4}$$

$$x_{h+1}^r(\delta^+(i)) - x_h^r(\delta^-(i)) = 0 \quad \forall r \in R, \quad \forall i \in R, \quad i \neq r,\ 0 \le h \le \theta - 1, \tag{5}$$

  Equation (4) insures that a total flow with index zero is generated at every relay node and is equal to the demand of that node. Equation (5) states that every relay node simply relays the information it receives and increments the hop count. Relay gateways do not have flow constraints because they act as sinks for the flow. The only constraints associated with relay gateways are capacity constraints. Note that this family of flow functions have zero values for most of the arcs in the graph with the exception of just a few.
- Capacity constraints,

$$\sum_{r \in R} \sum_{h=0}^{\theta} x_h^r(\delta^-(i)) + \beta(i) \le \gamma(i) \quad \forall i \in G \cup R. \tag{6}$$

  The constraint is a standard extension of capacity constraint in (3) for use with labelled flow functions.
- Delay constraints,

$$x_\theta^r(\delta^-(i)) = 0 \quad \forall i \in R. \tag{7}$$

  The relation makes sure that no relay node can receive flow of data that has reached the maximum delay threshold of θ. The flow of data must end at a relay gateway because otherwise flow constraint (5) is violated.

The objective function is again to minimize the cost of edges used in traffic weighted by the flow of data through the edges,

$$\min \sum_{h=0}^{\theta-1} \sum_{\substack{a \in A \\ r \in R}} c(a) \cdot x_h^r(a).$$

Using the graph in fig. 3(a) as input and choosing the upper bound on path length to be $\theta = 4$, the optimal solution (in fact, the only feasible solution) to the problem is depicted in fig. 4(a). The cost of the optimal solution is 8. The figure also displays some of the flow functions with non-zero values on the arcs of the graph.

Feasible paths model: This formulation is perhaps more direct than the one using flow functions because we assign flow of data to paths from RN to RG in the graph and not just to the arcs of the graph. Let r be a relay node and k a relay gateway in the description that follows.

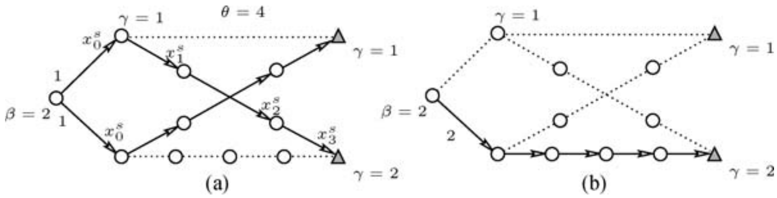


Fig. 4 (a) Optimal solution for the graph in fig. 3(a) given that the limit on path length is $\theta = 4$ edges; some of the arcs are labelled with the flow function with non-zero value on that arc. (b) Optimal solution for unconstrained single path problems for the graph in Fig. 3(a)

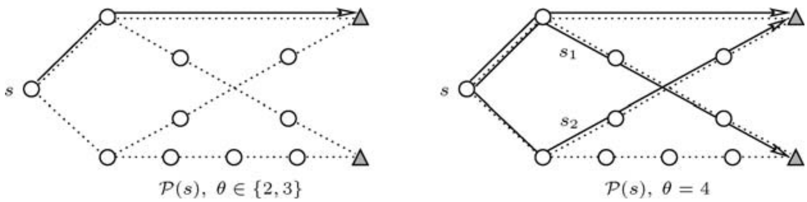


Fig. 5 Illustration of the set of feasible paths for graph in fig. 3(a)

Denote by $\mathcal{P}(r)$ the set of all directed paths in D, starting at r and ending at a relay gateway, with length no more than θ. Similarly, let $\mathcal{P}(g)$ be the set of path from any RN r to the given relay gateway g, and let $\mathcal{P}(r, g) = \mathcal{P}(r) \cap \mathcal{P}(g)$, i.e., the set of feasible paths from r to g. Let $\mathcal{P}$ be the set of all paths of length at most θ ending at a relay gateway. Figure 5 shows the set of feasible paths for the graph in fig. 3(a) for various values of delay bound θ. We use s to denote paths in the graph and we define function $x : \mathcal{P} \to \mathbb{N}$ that returns the value of data flow through a given path.

The delay constraints are now implicit; they are captured in the structure of set $\mathcal{P}$. The only constraints needed are those that insure the demand of every relay node is met (8) and that the capacity of every node is not violated (9).

$$\sum_{s \in \mathcal{P}(i)} x(s) = \beta(i) \quad \forall i \in R \tag{8}$$

$$\sum_{\substack{s \in \mathcal{P} \setminus \mathcal{P}(i) \\ i \in s}} x(s) + \beta(i) \leq \gamma(i) \quad \forall i \in G \cup R \tag{9}$$

The objective function here is to minimize the lengths of all feasible paths weighted by the data flow through the paths,

$$\min \sum_{s \in \mathcal{P}} c(s) \cdot x(s),$$

where $c(s)$ is the total cost of the path.

The optimal solution of example in fig. 3(a) for this model is, of course, identical with the one shown in fig. 4(a). The only difference lies in how the optimal solution is encoded. Here, the solution consists of assigning a flow of 1 for paths s_1 and s_2 in fig. 5.

2.3. Unconstrained single path relaying

In this problem, data originating at a particular RN must reach exactly one RG via a unique path. Figure 4(b) depicts the optimal solution for the graph in fig. 3(a). Only the relay gateway on the bottom can receive the flow of data since the capacity of the other gateway is insufficient, and there is only one feasible path that can support the traffic. The cost of the optimal solution is 10. In the following paragraphs we review the inequalities that define the problem in both the flow functions and feasible paths formulations.

Flow functions model: Since we need to make sure that data from a RN is not split among several paths, we use normalized flow functions. This means that the function takes values between 0 and 1. Intuitively, normalized flow functions represent the fraction of the total amount of data streaming out of a relay node. Let $y^r : A \to \{0, 1\}$ denote the normalized flow function for data generated at RN r.

We rewrite Eq. (3) to use normalised flow,

$$y^r(\delta^+(i)) - y^r(\delta^-(i)) = \begin{cases} 1, & \text{if } i = r \\ 0, & \text{otherwise} \end{cases} \quad \forall r \in R,\ i \in R.$$

The capacity constraint becomes

$$\sum_{r \in R \setminus \{i\}} \beta(r) y^r(\delta^-(i)) + \beta(i) \le \gamma(i) \quad \forall i \in G \cup R. \tag{10}$$

As in (3), we do not have flow conservation constraints at the relay gateways. The objective function becomes,

$$\min \sum_{\substack{r \in R \\ a \in A}} \beta(r) \cdot c(a) \cdot y^r(a).$$

Feasible paths model: The formulation based on enumeration of paths is an obvious extension of the path model in the previous section. We normalise the flow on paths and write demand and capacity constraints similar to (8) and (9). Using notation similar to that from Section 2.2, we define sets $\mathcal{P}$, $\mathcal{P}(r)$, $\mathcal{P}(g)$, and $\mathcal{P}(r, g)$ without length restriction and the normalised path flow function $y : \mathcal{P} \to \{0, 1\}$. First, we impose the demand constraints by assigning a total flow equal to 1 for all paths originating at the same relay node. Then, we write the capacity constraints.

$$\sum_{s \in \mathcal{P}(i)} y(s) = 1 \quad \forall i \in R \tag{11}$$

$$\sum_{\substack{r \in R \\ i \neq r}} \left(\beta(r) \sum_{s \in \mathcal{P}(r)} y(s) \right) + \beta(i) \le \gamma(i) \quad \forall i \in G \cup R \tag{12}$$

The objective function here is

$$\min \sum_{r\in R} \sum_{s\in\mathcal{P}(r)} \beta(r) \cdot c(s) \cdot y(s).$$

2.4. Single path relaying with constrained delay

Problem SPD combines the restrictions from the last two formulations, i.e. both the length of the path is restricted and the relaying path is unique. For the model using feasible paths, we can simply use Eqs. (11)–(12) with the definition of sets $\mathcal{P}$ from Section 2.1. For the model with flow functions, we adapt the hop based relations (4)–(7) to use normalised flow:

$$y_0^r(\delta^+(r)) = 1 \quad \forall r \in R, \tag{13}$$

$$y_{h+1}^r(\delta^+(i)) - y_h^r(\delta^-(i)) = 0$$
$$\forall r \in R, \quad \forall i \in R, \quad i \neq r, \quad 0 \leq h \leq \theta - 1, \tag{14}$$

$$\sum_{r\in R} \sum_{h=0}^{\theta} \beta(r)\, y_h^r(\delta^-(i)) + \beta(i) \leq \gamma(i) \quad \forall i \in G \cup R, \tag{15}$$

$$y_\theta^r(\delta^-(i)) = 0 \quad \forall i \in R. \tag{16}$$

For the formulation with flow functions, there exists another set of constraints that uses fewer flow functions without labelling them by hop count. We can reduce in this way the number of flow functions by a factor equal to θ. It is sufficient to just use functions $y^r : A \to \{0, 1\}$ with relations (10), (10), and the simple length bound of

$$\sum_{a\in A} y^r(a) \leq \theta \quad \forall r \in R. \tag{17}$$

If we consider the graph in fig. 3(a) as input and if we let $\theta = 4$ as in the example from fig. 4(a), then Problem SPD is infeasible. The only relay gateway that can be selected is node h and the only path of length at most four to reach node h cannot relay a flow of data larger than one unit because of the capacity of node a.

3. Conclusion

In this paper we consider the problem of multi-path relaying in a three tier hierarchical sensor network. Our work provides, for the first time in this context, a theoretical foundation for the study of problems arising in the management of data flow. Our assumptions begin with a given WSNet topology presented as a directed graph. We model the energy consumption by the cumulative traffic handled by the nodes assuming a known flow of data collected from sensors. We describe a unified model that encompasses a several additional constraints on data. In this model, we formulate the problem of managing traffic in a hierarchical WSNet by describing four optimisation problems in graphs. Each of the problems corresponds to a particular combination of data constraints. Out of the four problems, the two which restrict the relaying paths to exactly one for each source are NP-hard, as they contain the "generalised assignment problem" (Sahni and Gonzalez, 1976) as a special case.

Our contribution is mainly to specify these problems in a rigorous mathematical framework in the hope to inspire research in this direction and to offer a unified context for the comparison of different approaches. As such, we are currently designing centralised algorithms for some of the more difficult of the four problems identified here (Benkoczi et al., 2005). Our algorithms are exact and rely on linear and integer programming techniques. For problems STSPD and STSP, we chose the formulation based on feasible paths because in practice, it proves to be more efficient with memory usage. We combine a column generation algorithm with a branch and price scheme to solve non-trivial instances on graphs with sizes of the order of hundreds of vertices.

For the future, we consider extending our framework to capture and combine node placement, life cycle management, and data traffic management. We expect that such an approach would be more efficient in terms of cost. In doing this, we will consider optimisation problems from the domain of Location Theory. It is hoped that, as a by-product of our work, we would discover new generalisations of some known facility location problems. Such results are of interest to the location science community as well, since they are in line with the recent advances on generalising the p-median and p-center problems in graphs (Tamir, 1998, 2001; Benkoczi, 2004).

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