Multi-Hop Capacity of MIMO-Multiplexing Relaying Systems

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***Abstract*—This paper derives the multi-hop capacity of OFDM-based MIMO-multiplexing relaying systems. MIMO-multiplexing relaying presents a spectrally efficient means of realizing mesh supports in wireless networks operating over licensed bands by providing separate links for access and mesh relaying services on the same broadband radio channel. We show that for an NxN MIMO-multiplexing relaying system with amplification factor α at relay nodes, R-hops relaying degrade the capacity by at most $-N\log_2\left(\alpha^{2R}/\left(1+\sum_{r=1}^{R}\alpha^{2r}N^r\right)\right)+RN\log_2(N)$ bits/sec/Hz. Therefore, greater capacity loss is experienced in MIMO-multiplexing relaying involving high order MIMO systems. We also illustrate that the capacity loss is independent of the OFDM configurations employed; thus network operators could employ higher OFDM configurations to compensate for data rate loss in access services when some of the MIMO-multiplexing links are dedicated to mesh relay. This pioneering analysis provides useful guidelines for network operators planning to employ MIMO-multiplexing option for mesh relay supports.**

***Index Terms*—Multi-hop capacity, WiMAX mesh networks, MIMO-multiplexing relaying, amplify-and-forward relaying, broadband wireless access.**

I. Introduction

MIMO transmissions have been embraced in upcoming wireless standards like the WiMAX system, as advanced physical layer technology to enhance link quality or capacity. The two popular options for MIMO transmissions are space-time coding and MIMO-multiplexing systems. Space-time codes improve link quality performance, while MIMO-multiplexing enhances the wireless channel capacity [5], [6], [13]. OFDM signalling is also a broadband physical layer technology in upcoming wireless systems like the WiMAX. A number of works have therefore examined the performance of OFDM-based MIMO transmission systems, in the context of single-hop wireless transmissions (e.g. [10], [20]). A few works have also considered MIMO relaying networks [7]- [9]. However, available analyses are either applicable only to 2-hop wireless transmission systems (source-relay-destination), or the analyses are not done for OFDM-based MIMO transmission systems. In any case, the capacity of such systems has not, to our knowledge, been characterized as presented in this paper.

E. Telatar in his seminal work in [6] derived the single-hop capacity of MIMO wireless channels, where it was shown that the capacity grows linearly with the number of antennas. In this paper, we develop for the first time in the literature, the multi-hop extension of the work in [6]. This enables us to analyze the effects of multi-hop relaying on the MIMO capacity.

Applications: As illustrated in [1], for wireless networks like the WiMAX system where operation over licensed spectrum has been actively pursued [2], MIMO-multiplexing option at each node in the network can be employed to provide separate links for access and mesh relaying services on the same radio channel. In contrast with existing 802.11 WLAN mesh solution which provides separate radios, 2.4GHz and 5GHz radios respectively, for access and mesh services [4], MIMO-multiplexing mesh solution allows mesh and access services to be supported on the same broadband radio, providing a cost-effective means of realizing mesh supports, since operators can keep existing numbers of customers (and thus revenues or system capacity) for access services while providing mesh supports using this option for mesh services. Other applications of MIMO-multiplexing relaying include the IEEE 802.16j, the IEEE 802.11s [3], and the multi-hop cellular networks [17]-[19]. To illustrate the performance of the system in general applications, we derive expressions for the multi-hop capacity of OFDM-based MIMO-multiplexing relaying system, and observe the asymptotic effects of multi-hop relaying on the MIMO capacity.

II. Capacity Analysis

In this section, we characterize the achievable data rates over the multi-hop MIMO relaying channels. First we introduce the concept of 'multi-hop capacity'. Multi-hop capacity is the maximum mutual information between the source and destination node, over the multi-hop relaying channel [14]. Thus, for the application of MIMO-multiplexing relaying in wireless networks, we present next the multi-hop capacity of MIMO-multiplexing relaying system.

A. System model

We consider a MIMO-OFDM system employing n sub-carriers per OFDM symbol, M transmitting antennas and L receiving antennas. We assume that all M transmitting antennas at each node in a wireless network simultaneously

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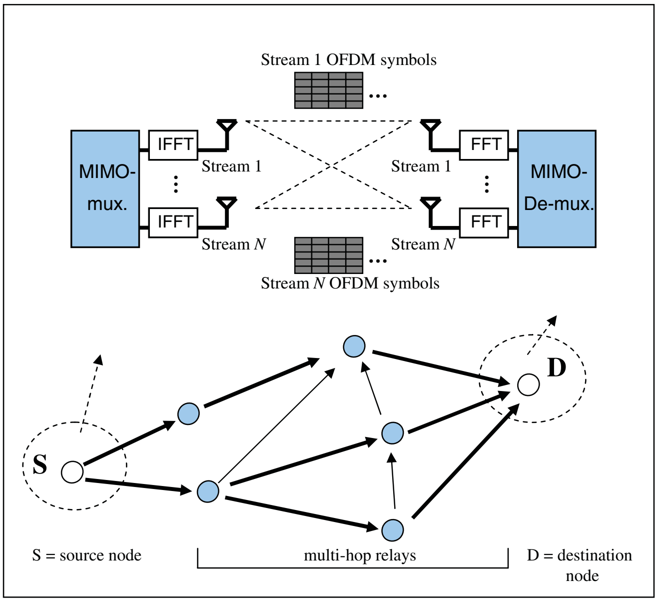

Fig. 1. MIMO-multiplexing relaying in broadband wireless networks.

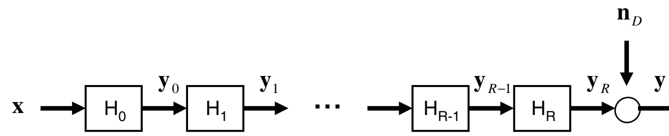

Fig. 2. Multi-hop channel model for AF MIMO-multiplexing relaying.

transmit different OFDM symbols, each of which can be assigned to either access or mesh relay stream, and that all transmissions are on the same frequency band. At the receiver, the space-multiplexed (access and mesh relay) streams are separated by tuning each of the L receiving antennas to the M transmitted streams. For mesh support, we assume that N transmitting and N receiving antennas (where $N \leq M, L$) are dedicated to the mesh streams at every node in the network. Thus, in the ensuing analysis we focus on NxN MIMO-multiplexing relaying system as illustrated in Fig 1.

1) Multi-hop channel model: Consider a wireless relaying system where the transmitted signal from a source node passes through a single-hop MIMO channel $\mathbf{H}_0$, and R multi-hop MIMO relaying channels $\mathbf{H}_1, \cdots, \mathbf{H}_R$, via R mesh relaying nodes, to the destination node. The MIMO channel matrix for each i-th hop transmission $\mathbf{H}_i, i = 0, 1, \cdots, R$, is an nNxnN block diagonal matrix, with the k^{th} block diagonal entries $\mathbf{H}[k]_i$ corresponding to the fading on the k^{th} OFDM subcarrier, $k = 0, \cdots, n-1$, modeled as independent and identically distributed (iid) random variables taken from zero-mean complex Gaussian distribution, with unit variance. We assume that the set $\left\{\mathbf{H}_0, \cdots, \mathbf{H}_R\right\}$ are independent, and that the Amplify-and-Forward (AF) relay option is employed at relay nodes, where relay nodes simply amplify and forward the OFDM symbols at the RF (radio frequency) stage, without decoding the contents. All relay nodes employ amplifiers with gain (or amplification factor) α. Network operators determine the values of α. Fig. 2 displays the equivalent model we have assumed for the resulting MIMO multi-hop channel [14], where $\mathbf{y}_i, i = 0, 1, \cdots, R$ represents the output of the i^{th} hop channel, and $\mathbf{y}_D$ represents the demodulated signal at the output of the FFT at the destination node. $\mathbf{y}_D$ is modeled as

$$\mathbf{y}_D = \alpha^R \Big(\mathbf{H}_R \mathbf{H}_{R-1} \cdots \mathbf{H}_1 \mathbf{H}_0\Big)\mathbf{x} + \mathbf{n}_D \tag{1}$$

where $\mathbf{x} = \left[x_0^0 x_0^1 \cdots x_0^{N-1} x_1^0 x_1^1 \cdots x_1^{N-1} \ \cdots \ x_{n-1}^0 x_{n-1}^1 \cdots x_{n-1}^{N-1}\right]^T$ is the nNx1 vector of the transmitted signals from the source node, and $\mathbf{n}_D$ is an nN x 1 complex additive noise vector that captures the over-all noise in the multi-hop channel. $\mathbf{n}_D$ is modeled as

$$\mathbf{n}_D = \mathbf{n}_0 + \mathbf{n}_1 + \cdots + \mathbf{n}_R \tag{2}$$

where $\mathbf{n}_R = \tilde{\mathbf{n}}_R$, $\mathbf{n}_{R-1} = \alpha \mathbf{H}_R \tilde{\mathbf{n}}_{R-1}$, $\mathbf{n}_{R-2} = \alpha^2 \mathbf{H}_R \mathbf{H}_{R-1} \tilde{\mathbf{n}}_{R-2}$, $\cdots$, $\mathbf{n}_1 = \alpha^{R-1} \mathbf{H}_R \cdots \mathbf{H}_2 \tilde{\mathbf{n}}_1$, $\mathbf{n}_0 = \alpha^R \mathbf{H}_R \cdots \mathbf{H}_1 \tilde{\mathbf{n}}_0$, and $\tilde{\mathbf{n}}_i$ is the nNx1 iid zero-mean complex AWGN vector, introduced at the i^{th} hop transmission on the n subcarriers. We assume that $E\left[\tilde{\mathbf{n}}_i \tilde{\mathbf{n}}_i^H\right] = \text{diag}\left\{\sigma^2 \mathbf{I}[k]_N\right\}_{k=0}^{n-1}$, $i = 0, \cdots, R$, where $\mathbf{I}[k]_N$ is an NxN identity matrix, and σ^2 is the variance of each elements of $\tilde{\mathbf{n}}_i$. For large N or R, the distribution of $\mathbf{n}_D$ can be approximated as Gaussian, using the central limit theory [21]. Since $E\left[\tilde{\mathbf{n}}_i\right] = \mathbf{0}$ for all i and $\tilde{\mathbf{n}}_i$ is independent of $\mathbf{H}_k$ for all i and k, we have that $E\left[\mathbf{n}_i\right] = \mathbf{0}$ for all i. Furthermore, since $\tilde{\mathbf{n}}_i$ and $\tilde{\mathbf{n}}_j$ are independent for $i \neq j$, we have for $i \neq j$ that $E\left[\mathbf{n}_i \mathbf{n}_j^H | \mathbf{H}_1, \cdots, \mathbf{H}_R\right]$ is identically the zero matrix, so by unconditioning we get $E\left[\mathbf{n}_i \mathbf{n}_j^H\right] = \mathbf{0}$. Thus we compute the covariance matrix of $\mathbf{n}_D$ as

$$E\left[\mathbf{n}_D \mathbf{n}_D^H\right] = E\left[\mathbf{n}_0 \mathbf{n}_0^H\right] + E\left[\mathbf{n}_1 \mathbf{n}_1^H\right] + \cdots + E\left[\mathbf{n}_{R-1} \mathbf{n}_{R-1}^H\right] + E\left[\mathbf{n}_R \mathbf{n}_R^H\right]. \tag{3}$$

It is straightforward to see that $E\left[\mathbf{H}_i \mathbf{H}_i^H\right] = N\mathbf{I}_{nN}$, for all i, since in our channel model, $E\left[|h_{l,m}^i|^2\right] = 1$ for the (l,m)th entry of $\mathbf{H}[k]_i$, $k = 0, 1, \cdots, n-1$, the block diagonal entries of the i^{th} hop MIMO channel $\mathbf{H}_i$, $i = 0, 1, \cdots, R$. Now, $E\left[\mathbf{H}_R \cdots \mathbf{H}_{i+1} \mathbf{H}_{i+1}^H \cdots \mathbf{H}_R^H\right] = N^{R-i}\mathbf{I}_{nN}$, by Lemma 1 in Appendix-A, for all $i = 0, \cdots, R-1$, so by conditioning on $\mathbf{H}_1, \cdots, \mathbf{H}_R$, we have that $E\left[\mathbf{n}_i \mathbf{n}_i^H | \mathbf{H}_1, \cdots, \mathbf{H}_R\right] = \sigma^2 \alpha^{2(R-i)} \mathbf{H}_R \cdots \mathbf{H}_{i+1} \mathbf{H}_{i+1}^H \cdots \mathbf{H}_R^H$. Unconditioning then gives $E\left[\mathbf{n}_i \mathbf{n}_i^H\right] = \sigma^2 \alpha^{2(R-i)} N^{R-i} \mathbf{I}_{nN}$. Therefore,

$$E\left[\mathbf{n}_D \mathbf{n}_D^H\right] = \sigma^2 \left(1 + \sum_{r=1}^{R} \alpha^{2r} N^r\right) \mathbf{I}_{nN}. \tag{4}$$

Since the covariance matrix of $\mathbf{n}_D$ is diagonal, we conclude that the elements of $\mathbf{n}_D$ are iid [21, pp. 257].

B. Derivation of the multi-hop Capacity

Given that $\mathbf{n}_D$ is an iid zero-mean complex Guassian noise as shown in Eq. (2)-(4), and let $\mathbf{x}$ be zero-mean, circularly symmetric complex Gaussian input data, with covariance

$E\left[\mathbf{x}\mathbf{x}^{H}\right] = \mathbf{G}$. Then the entropy of $\mathbf{y}_D$ is the largest, and the maximum mutual information between the input and the output of the multi-hop MIMO-OFDM channel is given by [12]

$$\begin{aligned} &I\left(\mathbf{x};\left(\mathbf{y}_D,\mathbf{H}_R,\cdots,\mathbf{H}_0\right)\right)\\ &\quad= I\left(\mathbf{x};\mathbf{H}_R,\cdots,\mathbf{H}_0\right) + I\left(\mathbf{x};\mathbf{y}_D|\mathbf{H}_R,\cdots,\mathbf{H}_0\right)\\ &\quad= I\left(\mathbf{x};\mathbf{y}_D|\mathbf{H}_R,\cdots,\mathbf{H}_0\right)\\ &\quad= \frac{1}{n}E_{\mathbf{H}_R,\cdots,\mathbf{H}_0}\Big[\log_2\Big\{\det\Big[\mathbf{I}_{nN} + \frac{\alpha^{2R}}{\sigma^2_{n_D}}\\ &\quad\cdot\Big(\mathbf{H}_R\mathbf{H}_{R-1}\cdots\mathbf{H}_1\mathbf{H}_0\Big)\mathbf{G}\Big(\mathbf{H}_R\mathbf{H}_{R-1}\cdots\mathbf{H}_1\mathbf{H}_0\Big)^{H}\Big]\Big\}\Big], \end{aligned} \tag{5}$$

where $\sigma^2_{n_D} = \sigma^2\Big(1 + \sum_{r=1}^{R}\alpha^{2r}N^r\Big)$. Notice that $\mathbf{G}$ is an nNxnN non-negative matrix with block-diagonal entries given by $\mathbf{G}_k = E\Big\{\left[x_k^0 x_k^1\cdots x_k^{N-1}\right]\left[x_k^0 x_k^1\cdots x_k^{N-1}\right]^{H}\Big\}$, $k = 0,1,\cdots,n-1$. When the channel state information (CSI) is not available at the transmitter, the total available power P, is allocated uniformly across all space-frequency subchannels, which corresponds to setting $\mathbf{G}_k = \frac{P}{nN}\mathbf{I}_N$, for $k = 0,1,\cdots,n-1$. Therefore, the ergodic capacity of the multi-hop MIMO-OFDM channel is given by

$$\begin{aligned} \mathcal{C} = &\frac{1}{n}E_{\mathbf{H}_R,\cdots,\mathbf{H}_0}\Big[\log_2\Big\{\det\Big[\mathbf{I}_{nN} + \frac{\alpha^{2R}}{\sigma^2_{n_D}}\\ &\cdot\frac{P}{nN}\Big(\mathbf{H}_R\mathbf{H}_{R-1}\cdots\mathbf{H}_1\mathbf{H}_0\Big)\Big(\mathbf{H}_R\mathbf{H}_{R-1}\cdots\mathbf{H}_1\mathbf{H}_0\Big)^{H}\Big]\Big\}\Big] \end{aligned} \tag{6}$$

which can be written as

$$\begin{aligned} \mathcal{C} = \frac{1}{n}\sum_{k=0}^{n-1} &E_{\mathbf{H}[k]_R,\cdots,\mathbf{H}[k]_0}\Big[\log_2\Big\{\det\Big[\mathbf{I}_N + \rho\\ &\cdot\Big(\mathbf{H}[k]_R\mathbf{H}[k]_{R-1}\cdots\mathbf{H}[k]_1\mathbf{H}[k]_0\Big)\\ &\cdot\Big(\mathbf{H}[k]_R\mathbf{H}[k]_{R-1}\cdots\mathbf{H}[k]_1\mathbf{H}[k]_0\Big)^{H}\Big]\Big\}\Big] \end{aligned} \tag{7}$$

where

$$\rho = \frac{\alpha^{2R}P}{nN\sigma^2_{n_D}}. \tag{8}$$

Notice that setting $R = 0$ and $n = 1$, Eq. (7) reduces to the single-hop, single-carrier transmission case. Using $|\mathbf{I}+\mathbf{XY}| = |\mathbf{I}+\mathbf{YX}|$, the expression for the ergodic multihop capacity can be written as

$$\begin{aligned} \mathcal{C} &= \frac{1}{n}\sum_{k=0}^{n-1}E_{\mathbf{H}[k]_R,\cdots,\mathbf{H}[k]_0}\Big[\log_2\Big\{\det\Big[\mathbf{I}_N + \rho\\ &\quad\cdot\mathbf{H}[k]_R\mathbf{H}[k]_R^{H}\mathbf{H}[k]_{R-1}\mathbf{H}[k]_{R-1}^{H}\cdots\mathbf{H}[k]_0\mathbf{H}[k]_0^{H}\Big]\Big\}\Big]\\ &= \frac{1}{n}\sum_{k=0}^{n-1}E_{\mathbf{H}[k]_R,\cdots,\mathbf{H}[k]_0}\Big[\log_2\Big\{\det\Big[\mathbf{I}_N + \rho\\ &\quad\cdot\mathbf{Q}_{k,R}\mathbf{Q}_{k,R-1}\cdots\mathbf{Q}_{k,0}\Big]\Big\}\Big] \end{aligned} \tag{9}$$

where $\mathbf{Q}_{k,j} = \mathbf{H}[k]_j\mathbf{H}[k]_j^{H}$.

1) Evaluation of the multi-hop capacity: Next we employ eigenvalue decomposition of $\mathbf{Q}_{k,i}$, $i = 0,1,\cdots,R$. $\mathbf{Q}_{k,i}$, $i = 0,1,\cdots,R$, are NxN random non-negative definite matrix and have real non-negative eigenvalues. We can therefore express the ergodic multihop capacity in terms of the eigen values $\lambda_1^{k,i},\cdots,\lambda_N^{k,i}$ of $\mathbf{Q}_{k,i}$, $i = 0,1,\cdots,R$ as

$$\mathcal{C} = \frac{1}{n}\sum_{k=0}^{n-1}\sum_{i=1}^{N}E_{\lambda_i^{k,R},\cdots,\lambda_i^{k,0}}\Big[\log_2\Big(1 + \rho\prod_{j=0}^{R}\lambda_i^{k,j}\Big)\Big], \tag{10}$$

which can be written as

$$\mathcal{C} = \frac{1}{n}\sum_{k=0}^{n-1}NE_{\lambda^{k,R},\cdots,\lambda^{k,0}}\Big[\log_2\Big(1 + \rho\prod_{i=0}^{R}\lambda^{k,i}\Big)\Big], \tag{11}$$

where $\lambda^{k,i}$ is a randomly selected eigenvalue from the eigenvalues $\Big\{\lambda_1^{k,i},\cdots,\lambda_N^{k,i}\Big\}$ of $\mathbf{Q}_{k,i}$, $i = 0,1,\cdots,R$. The pdf of $\lambda^{k,i}$ obtained in [6] can be expressed for our case here as

$$f_{\lambda^{k,i}}(\lambda) = \frac{1}{N}\sum_{k=0}^{N-1}\Big[L_k^0(\lambda)\Big]^2 e^{-\lambda}, \tag{12}$$

where $L_k^0(\lambda)$ is the associated Laguerre polynomial of order k, which can be expressed in series form as [15],

$$L_k^0(\lambda) = \sum_{l=0}^{k}(-1)^l\frac{k!}{(k-l)!l!l!}\lambda^l \tag{13}$$

Inserting Eq. (13) into the pdf in Eq. (12) gives:

$$f_{\lambda^{k,i}}(\lambda) = \frac{1}{N}\sum_{k=0}^{N-1}\Big(\sum_{l_1=0}^{k}\sum_{l_2=0}^{k}(-1)^{l_1+l_2}A_{l_1}(k)A_{l_2}(k)\lambda^{l_1+l_2}\Big)e^{-\lambda} \tag{14}$$

where $A_l(k) = \frac{k!}{(k-l)!l!l!}$.
Using the iid assumption of the multi-hop MIMO channel matrices, then $\mathbf{Q}_{k,i}$, $i = 0,1,\cdots,R$ are iid. Thus $\lambda^{k,i}$, $i = 0,1,\cdots,R$, are also iid. Therefore the ergodic multi-hop capacity in Eq. (11) can be expressed as

$$\begin{aligned} \mathcal{C} &= \frac{N}{n}\sum_{k=0}^{n-1}\underbrace{\int_0^\infty\cdots\int_0^\infty}_{(R+1)-\text{fold}}\Big[\log_2\Big(1+\rho\prod_{i=0}^{R}\lambda^{k,i}\Big)\Big]\\ &\quad\cdot f_{\lambda^{k,R},\cdots,\lambda^{k,0}}(\lambda^{k,R},\cdots,\lambda^{k,0})\,d\lambda^{k,R}\cdots d\lambda^{k,0}\\ &= \frac{N}{n}\sum_{k=0}^{n-1}\underbrace{\int_0^\infty\cdots\int_0^\infty}_{R-\text{fold}}\Big(\frac{1}{N}\sum_{q_0=0}^{N-1}\sum_{l_1^0=0}^{q_0}\sum_{l_2^0=0}^{q_0}(-1)^{l_1^0+l_2^0}A_{l_1^0}(q_0)\\ &\quad\cdot A_{l_2^0}(q_0)\int_0^\infty\log_2\Big(1+\rho\prod_{i=0}^{R}\lambda^{k,i}\Big)\Big(\lambda^{k,0}\Big)^{l_1^0+l_2^0}e^{-\lambda^{k,0}}\,d\lambda^{k,0}\Big)\\ &\quad\cdot\prod_{j=1}^{R}\Big(\frac{1}{N}\sum_{q_j=0}^{N-1}\sum_{l_1^j=0}^{q_j}\sum_{l_2^j=0}^{q_j}(-1)^{l_1^j+l_2^j}A_{l_1^j}(q_j)\\ &\quad\cdot A_{l_2^j}(q_j)\Big(\lambda^{k,j}\Big)^{l_1^j+l_2^j}e^{-\lambda^{k,j}}\Big)\,d\lambda^{k,R}\cdots d\lambda^{k,1} \end{aligned} \tag{15}$$

The inner integral in (15) can be represented as $I_{l_1^0+l_2^0}(a) = \int_0^\infty\log_2\Big(1 + a\lambda^{k,0}\Big)\left(\lambda^{k,0}\right)^{l_1^0+l_2^0}e^{-\lambda^{k,0}}\;d\lambda^{k,0}$, where $a =$

$\rho\prod_{i=1}^{R}\lambda^{k,i}$. Integrating this integral by parts $(l_1^0+l_2^0)$ times yields [13], [15]:

$$I_{l_1^0+l_2^0}(a)=\sum_{\mu=0}^{l_1^0+l_2^0}\frac{(l_1^0+l_2^0)!}{(l_1^0+l_2^0-\mu)!}\Big[(-1)^{l_1^0+l_2^0-\mu-1}(1/a)^{l_1^0+l_2^0-\mu}\cdot e^{\frac{1}{a}}\mathrm{Ei}(-1/a)+\sum_{k=1}^{l_1^0+l_2^0-\mu}(k-1)!(-1/a)^{l_1^0+l_2^0-\mu-k}\Big] \tag{16}$$

where $\mathrm{Ei}(x)=\int_{-\infty}^{x}\frac{e^t}{t}\,dt$ is the exponential integral. Direct substitution of Eq. (16) into (15) results in an expression not readily expressed in closed-form due to the presence of the $\mathrm{Ei}(x)$ function under the integral. We thus express $\mathrm{Ei}(x)$ using the series representation [15]

$$\mathrm{Ei}(x)=b+\ln(-x)+\sum_{k=1}^{\infty}\frac{x^k}{k\cdot k!}\qquad x<0,\ b\approx 0.577, \tag{17}$$

and employ the following approximation, useful in high SNR (i.e. when $\rho\gg 0$, in which case $1/a\ll 1$):

$$\mathrm{Ei}(x)\approx b+\ln(-x)\quad |x|\ll 1. \tag{18}$$

Using this approximation, Eq. (16) can be simplified asymptotically as

$$I_{l_1^0+l_2^0}(a)\approx(l_1^0+l_2^0)!\left[\ln(a)-b+\sum_{\mu=1}^{l_1^0+l_2^0}\frac{1}{\mu}\right]. \tag{19}$$

Substituting (19) into (15) yields:

$$\begin{aligned}\mathcal{C}\approx\frac{N}{n}\sum_{k=0}^{n-1}\underbrace{\int_0^\infty\cdots\int_0^\infty}_{R-\text{fold}}&\Bigg(\frac{1}{N}\sum_{q_0=0}^{N-1}\sum_{l_1^0=0}^{q_0}\sum_{l_2^0=0}^{q_0}(-1)^{l_1^0+l_2^0}A_{l_1^0}(q_0)\\&\cdot A_{l_2^0}(q_0)(l_1^0+l_2^0)!\left[\ln(a)-b+\sum_{\mu=1}^{l_1^0+l_2^0}\frac{1}{\mu}\right]\Bigg)\\&\cdot\prod_{j=1}^{R}\Bigg(\frac{1}{N}\sum_{q_j=0}^{N-1}\sum_{l_1^j=0}^{q_j}\sum_{l_2^j=0}^{q_j}(-1)^{l_1^j+l_2^j}A_{l_1^j}(q_j)\\&\cdot A_{l_2^j}(q_j)\left(\lambda^{k,j}\right)^{l_1^j+l_2^j}e^{-\lambda^{k,j}}\Bigg)\,d\lambda^{k,R}\cdots d\lambda^{k,1}.\end{aligned} \tag{20}$$

In Appendix-B, we show that Eq. (20) has a closed-form solution given by Eq. (21). For the case of 2x2 MIMO-multiplexing relaying (i.e., $N=2$), Eq. (21) reduces to

$$\mathcal{C}\approx 2\Big(\ln(\rho)-b(R+1)+(\frac{1}{2}R+\frac{1}{2})\Big). \tag{22}$$

2) Effect of fading correlation on the multi-hop MIMO capacity: Assuming that the MIMO channel sub-matrix for each OFDM sub-carrier at the j^{th} hop transmission, has correlated elements and can be expressed as

$$\tilde{\mathbf{H}}[k]_j=\mathbf{R}_{Rec,j}^{1/2}\mathbf{H}[k]_j\mathbf{R}_{Tx,j}^{1/2},\quad j=0,1,\cdots,R \tag{23}$$

where $\mathbf{R}_{Tx,j}$ and $\mathbf{R}_{Rec,j}$ are positive definite Hermitian matrices modeling fading correlations at the transmitter and receiver side respectively. The elements of $\mathbf{R}_{Tx,j}$ and $\mathbf{R}_{Rec,j}$ are normalized. Thus, the multi-hop capacity for correlated fading is given by

$$\begin{aligned}\mathcal{C}&=\frac{1}{n}\sum_{k=0}^{n-1}E_{\mathbf{H}[k]_R,\cdots,\mathbf{H}[k]_0}\Bigg[\log_2\Bigg\{\det\Bigg[\mathbf{I}_N+\rho\mathbf{R}_{Rec,R}^{1/2}\\&\cdot\mathbf{H}[k]_R\mathbf{R}_{Tx,R}\mathbf{H}[k]_R^H(\mathbf{R}_{Rec,R}^{1/2})^H\cdots\\&\cdot\mathbf{R}_{Rec,0}^{1/2}\mathbf{H}[k]_0\mathbf{R}_{Tx,0}\mathbf{H}[k]_0^H(\mathbf{R}_{Rec,0}^{1/2})^H\Bigg]\Bigg\}\Bigg]\\&=\frac{1}{n}\sum_{k=0}^{n-1}E_{\mathbf{H}[k]_R,\cdots,\mathbf{H}[k]_0}\Bigg[\log_2\Bigg\{\det\Bigg[\mathbf{I}_N+\rho(\mathbf{R}_{Tx,R}\mathbf{R}_{Rec,R}^H)\\&\cdot\mathbf{H}[k]_R\mathbf{H}[k]_R^H\cdots(\mathbf{R}_{Tx,0}\mathbf{R}_{Rec,0}^H)\mathbf{H}[k]_0\mathbf{H}[k]_0^H\Bigg]\Bigg\}\Bigg].\end{aligned} \tag{24}$$

At asymptotically high SNR, the capacity can be approximated as

$$\begin{aligned}\mathcal{C}\approx&\frac{1}{n}\sum_{k=0}^{n-1}E_{\mathbf{H}[k]_R,\cdots,\mathbf{H}[k]_0}\Big[\log_2\Big\{\det\Big[\rho\mathbf{H}[k]_R\mathbf{H}[k]_R^H\\&\cdots\mathbf{H}[k]_0\mathbf{H}[k]_0^H\Big]\Big\}\Big]\\&+\sum_{j=0}^{R}\Bigg(\log_2\{\det[\mathbf{R}_{Rec,j}]\}+\log_2\{\det[\mathbf{R}_{Tx,j}]\}\Bigg).\end{aligned} \tag{25}$$

For the case of N x N MIMO-multiplexing relaying considered here, $\mathbf{R}_{Tx,j}$ and $\mathbf{R}_{Rec,j}$ are full rank, $j=0,1,\cdots,R$ [16]. Denote the eigen values of $\mathbf{R}_{Rec,j}$ by λ_i^j, $i=1,\cdots,N$, $j=0,1,\cdots,R$. From the arithmetic mean-geometric mean inequality [16], $\prod_{i=1}^{N}\lambda_i^j\le 1$. Since $(\mathbf{R}_{Rec,j})=\prod_{i=1}^{N}\lambda_i^j$, it follows that $\log_2\det(\mathbf{R}_{Rec,j})\le 0$ and is zero only if all eigen values of $\mathbf{R}_{Rec,j}$ are equal, i.e., $\mathbf{R}_{Rec,j}=\mathbf{I}_N$. A similar argument holds for $\mathbf{R}_{Tx,j}$, $j=0,1,\cdots,R$. Thus, we conclude that for MIMO-multiplexing relaying over a multi-hop channel, fading correlation has a cumulative detrimental effect on the capacity as more hops are traversed, and the loss in ergodic capacity at high SNR is given by

$$\begin{aligned}\mathcal{C}_{\text{loss,correlation}}(R)=\sum_{j=0}^{R}\Big(&\log_2\Big\{\det[\mathbf{R}_{Rec,j}]\Big\}\\&+\log_2\Big\{\det[\mathbf{R}_{Tx,j}]\Big\}\Big)\text{bps/Hz}.\end{aligned} \tag{26}$$

3) Asymptotic effect of multi-hop relaying on the MIMO capacity: At high SNR, the multi-hop capacity expression in Eq. (9) can be approximated as

$$\begin{aligned}\mathcal{C}\approx\frac{1}{n}\sum_{k=0}^{n-1}E_{\mathbf{H}[k]_R,\cdots,\mathbf{H}[k]_0}\Big[\log_2\Big\{\det\Big[\rho\mathbf{H}[k]_R\mathbf{H}[k]_R^H\\\cdots\mathbf{H}[k]_0\mathbf{H}[k]_0^H\Big]\Big\}\Big].\end{aligned}$$

Noting that $\det(AB)=\det(A)\cdot\det(B)$, and invoking the iid assumption of $\mathbf{H}[k]_i$, $i=0,1,\cdots,R$, we have

$$\mathcal{C} \approx \sum_{q_0=0}^{N-1}\sum_{l_1^0=0}^{q_0}\sum_{l_2^0=0}^{q_0}(-1)^{l_1^0+l_2^0}A_{l_1^0}(q_0)A_{l_2^0}(q_0)(l_1^0+l_2^0)! \cdot \Bigg\{\ln(\rho)\cdot\prod_{j=1}^{R}\Bigg(\frac{1}{N}\sum_{q_j=0}^{N-1}\sum_{l_1^j=0}^{q_j}\sum_{l_2^j=0}^{q_j}(-1)^{l_1^j+l_2^j}A_{l_1^j}(q_j) \cdot A_{l_2^j}(q_j)\left(l_1^j+l_2^j\right)!\Bigg) + \sum_{i=1}^{R}\Bigg[\Bigg(\frac{1}{N}\sum_{q_i=0}^{N-1}\sum_{l_1^i=0}^{q_i}\sum_{l_2^i=0}^{q_i}(-1)^{l_1^i+l_2^i}A_{l_1^i}(q_i) \cdot A_{l_2^i}(q_i)\Bigg(\sum_{\mu=0}^{(l_1^i+l_2^i-1)}\frac{(l_1^i+l_2^i)!}{(l_1^i+l_2^i-\mu)} - (l_1^i+l_2^i)!b\Bigg)\Bigg) \cdot \prod_{j=1,j\neq i}^{R}\Bigg(\frac{1}{N}\sum_{q_j=0}^{N-1}\sum_{l_1^j=0}^{q_j}\sum_{l_2^j=0}^{q_j}(-1)^{l_1^j+l_2^j}A_{l_1^j}(q_j)A_{l_2^j}(q_j)(l_1^j+l_2^j)!\Bigg)\Bigg] + (-b)\prod_{j=1}^{R}\Bigg(\frac{1}{N}\sum_{q_j=0}^{N-1}\sum_{l_1^j=0}^{q_j}\sum_{l_2^j=0}^{q_j}(-1)^{l_1^j+l_2^j}A_{l_1^j}(q_j)A_{l_2^j}(q_j)\left(l_1^j+l_2^j\right)!\Bigg) + \sum_{\mu=1}^{l_1^0+l_2^0}\frac{1}{\mu}\cdot\prod_{j=1}^{R}\Bigg(\frac{1}{N}\sum_{q_j=0}^{N-1}\sum_{l_1^j=0}^{q_j}\sum_{l_2^j=0}^{q_j}(-1)^{l_1^j+l_2^j}A_{l_1^j}(q_j) \cdot A_{l_2^j}(q_j)\left(l_1^j+l_2^j\right)!\Bigg)\Bigg\}. \tag{21}$$

$$\begin{aligned}\mathcal{C} &\approx \frac{1}{n}\sum_{k=0}^{n-1}E_{\mathbf{H}[k]_0}\Big[\log_2\Big\{\det\Big[\rho\mathbf{H}[k]_0\mathbf{H}[k]_0^H\Big]\Big\}\Big] + E_{\mathbf{H}[k]_1}\Big[\log_2\Big\{\det\Big[\mathbf{H}[k]_1\mathbf{H}[k]_1^H\Big]\Big\}\Big] + \cdots + E_{\mathbf{H}[k]_R}\Big[\log_2\Big\{\det\Big[\mathbf{H}[k]_R\mathbf{H}[k]_R^H\Big]\Big\}\Big] \\ &= \frac{1}{n}\sum_{k=0}^{n-1}E_{\mathbf{H}[k]_0}\Big[\log_2\Big\{\det\Big[\rho_0\mathbf{H}[k]_0\mathbf{H}[k]_0^H\Big]\Big\}\Big] + N\log_2(\rho_1) + \sum_{j=1}^{R}E_{\mathbf{H}[k]_j}\Big[\log_2\Big\{\det\Big[\mathbf{H}[k]_j\mathbf{H}[k]_j^H\Big]\Big\}\Big].\end{aligned} \tag{27}$$

where $\rho_0 = P/nN\sigma^2$, and $\rho_1 = \alpha^{2R}/(1+\sum_{r=1}^{R}\alpha^{2r}N^r)$. We notice that $\mathbf{H}[k]_j\mathbf{H}[k]_j^H$, $j = 1,\cdots,R$, all have positive, real eigenvalues [6]. For the case of N x N MIMO-multiplexing relaying considered here, $\mathbf{H}[k]_j\mathbf{H}[k]_j^H$, $j = 1,\cdots,R$, are all full rank. Let $\tilde{\lambda}_i^j$ denote the eigen values of $\mathbf{H}[k]_j\mathbf{H}[k]_j^H$, $j = 1,\cdots,R$, then for normalized channel, $\det\Big[\mathbf{H}[k]_j\mathbf{H}[k]_j^H\Big] = \prod_{j=1}^{N}\tilde{\lambda}_i^j \leq 1$. Also, $\rho_1 = \alpha^{2R}/(1+\sum_{r=1}^{R}\alpha^{2r}N^r) \leq 1$. Therefore, MIMO-multiplexing relaying over multi-hop channel reduces the MIMO capacity as more hops are traversed, and the loss in ergodic capacity at high SNR, can be expressed for each OFDM sub-carrier, k, as

$$\mathcal{C}_{loss,R-hop-relay}(R) = N\log_2\Big(\alpha^{2R}/(1+\sum_{r=1}^{R}\alpha^{2r}N^r)\Big) + \sum_{j=1}^{R}E_{\mathbf{H}[k]_j}\Big[\log_2\Big\{\det\Big[\mathbf{H}[k]_j\mathbf{H}[k]_j^H\Big]\Big\}\Big]\text{bps/Hz}. \tag{28}$$

For large N, $N\left(\frac{1}{N}\mathbf{H}[k]_j\mathbf{H}[k]_j^H\right) \rightarrow N\mathbf{I}_N$. Thus the loss in ergodic capacity, per subcarrier, is upper-bounded by

$$\mathcal{C}_{loss,R-hop-relay}(R) < -N\log_2\Big(\alpha^{2R}/(1+\sum_{r=1}^{R}\alpha^{2r}N^r)\Big) + RN\log_2(N)\text{bps/Hz}. \tag{29}$$

Given the fact that for large N, the capacity becomes insensitive to the realizations of $\mathbf{H}[k]_j$ [11], Eq. (29) gives the upper-bound on the loss in ergodic capacity due to multi-hop relaying, regardless of the specific realizations of the MIMO channel matrices.

Eq. (29) suggests that R-hops relaying contribute capacity degradation that is asymptotically upper-bounded by $-N\log_2\Big(\alpha^{2R}/(1+\sum_{r=1}^{R}\alpha^{2r}N^r)\Big)+RN\log_2(N)$ bps/Hz. In other words, high-order MIMO systems suffer greater capacity loss due to multi-hop relaying than low-order MIMO systems.

4) Data rate loss in access services with MIMO-multiplexing mesh supports: For M x L MIMO configuration, with N x N configuration ($N \leq M, L$) dedicated to mesh relaying supports at each node, the loss in data rate to access service per node can be estimated as [1, problem #9]:

$$\mathcal{C}_{loss,access} = m_1\log_2\Big(1+SNR\Big) - m_2\log_2\Big(1+SNR\Big), \tag{30}$$

where $m_1 = \min[M, L]$, and $m_2 = \min[(M-N),(L-N)]$. This loss can be compensated by switching to a higher OFDM configuration or increasing the number of excited subcarriers per OFDM symbol.

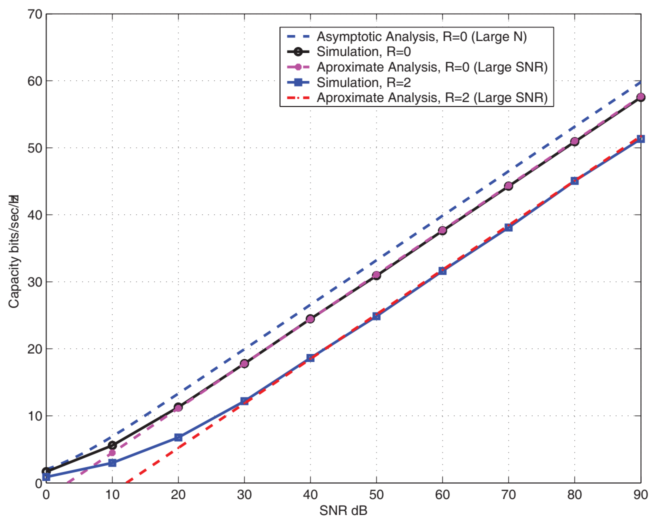

Fig. 3. Analytical and simulation results for the multi-hop capacity of 2x2 MIMO-multiplexing relaying

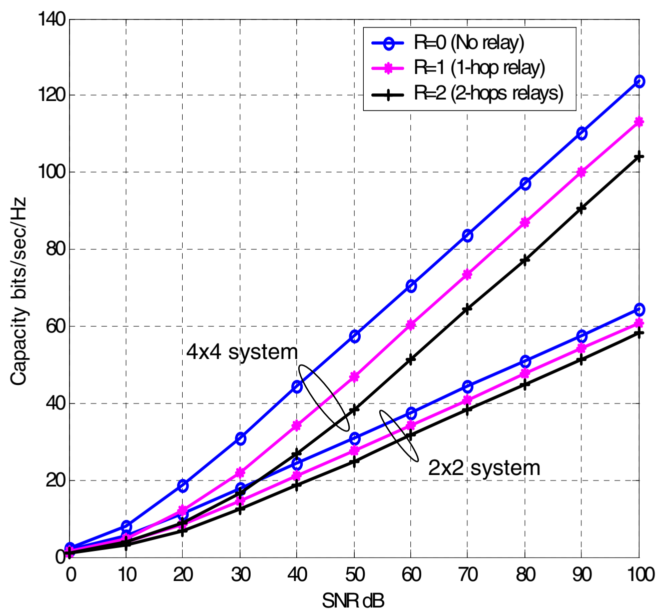

Fig. 4. Effect of multi-hop relaying on the capacity of MIMO-multiplexing relaying systems

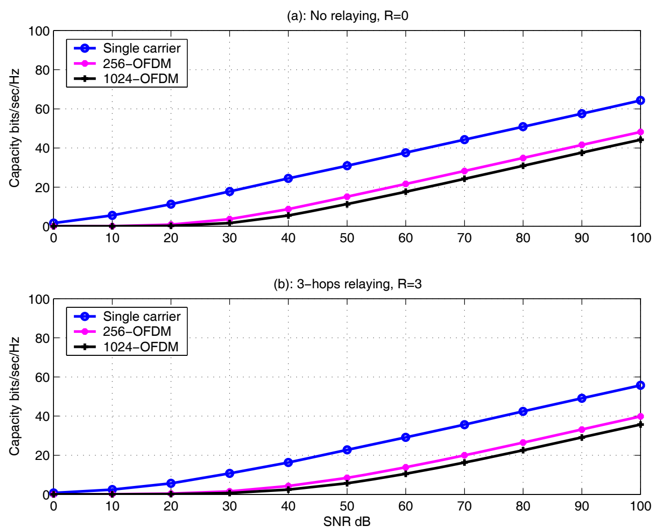

Fig. 5. Effect of multi-hop relaying on the capacity of different MIMO-OFDM systems, $R = 0$ and $R = 3$

III. Simulation Results

In this section, we present simulation results to complement the analysis. In our simulation model, we generate large realizations of the multihop MIMO channel gains, $\{\mathbf{H}[k]_0, \mathbf{H}[k]_1, \cdots, \mathbf{H}[k]_R\}$, for each OFDM subcarrier, k, $k = 0, 1, \cdots, n-1$. Each $\mathbf{H}[k]_j$, $j = 0, \cdots, R$, has iid random entries taken from complex Gaussian distribution. For each $\mathbf{H}[k]_j$, we compute the eigen values of $\mathbf{Q}_{k,j} = \mathbf{H}[k]_j \mathbf{H}[k]_j^H$, $j = 0, 1, \cdots, R$, and substitute them into Eq. (10). The expression is then averaged over the numbers of samples generated. First we examine the cases $n = 1$ and $\alpha = 1$. Fig 3 compares our simulation results for the multi-hop capacity of MIMO-multiplexing relaying system, with the approximate analysis. 2x2 MIMO-multiplexing relaying is assumed for the results in this figure. As expected, both results agree very well in high-SNR region. The capacity limit obtained using the assumption of asymptotically large antennas, N, in [6] is also included in this figure for comparison. Fig 4 presents the simulation results on the asymptotic effects of multi-hop relaying on MIMO channel capacity, for the 2x2 and 4x4 MIMO-multiplexing relaying configurations. It is observed from the results that the capacity of MIMO-multiplexing relaying with 1-hop relay or more, $R \geq 1$, degrades compared to the case of no relay ($R = 0$), and that the loss in capacity due to R-hops relaying is truly upper-bounded as in Eq. (29). For example for the cases $R = 1$, and $R = 2$ for 2x2 system in this figure, the capacity loss due to multihop relaying are respectively: $(64.3 - 60.8) \approx 4$ bits/sec/Hz, and $(64.3 - 58.2) \approx 6$ bits/sec/Hz, per OFDM subcarrier in high SNR, which are well upper-bounded by the respective values predicted by Eq. (29): $5.2 \approx 5$ bits/sec/Hz, and $8.6 \approx 9$ bits/sec/Hz. For cases $R = 1$, and $R = 2$ for the 4x4 system in the figure, the capacity loss are respectively: $(123.8 - 113.4) \approx 10$ bits/sec/Hz, and $(123.8 - 104) \approx 20$ bits/sec/Hz, per OFDM subcarrier in high SNR. Comparing these data with the counterpart data for the 2x2 system, we conclude that high-order MIMO-multiplexing relaying incures higher capacity loss, as predicted by Eq. (29). Next we examine the effects of multi-hop relaying on the capacity of different OFDM systems employed in broadband wireless networks such as the WiMAX system. In Fig 5, we present the simulation results for the multi-hop capacity of 2x2 MIMO-multiplexing relaying for single-carrier transmission, 256-OFDM, and 1024-OFDM systems, cases of $R = 0$ and $R = 3$. From this figure, we observe that at $SNR = 100$ dB, the capacity loss due to 3-hops relaying for the single-carrier transmission case is $(64.3 - 55.8) = 8.5$ bits/sec/Hz. For the multi-carrier transmission systems, the corresponding capacity losses for 256-OFDM, and 1024-OFDM are respectively $(48.2 - 39.8) = 8.4$ bits/sec/Hz, and $(44.2 - 35.8) = 8.4$ bits/sec/Hz. Therefore, we conclude from this data that the capacity loss due to multi-hop relaying is fairly constant across all OFDM systems. Finally, Fig 6 illustrates the effects of amplifier gains at relay nodes. It is observed from the results that higher amplifier gains can enhance the capacity, despite noise amplifications, because the

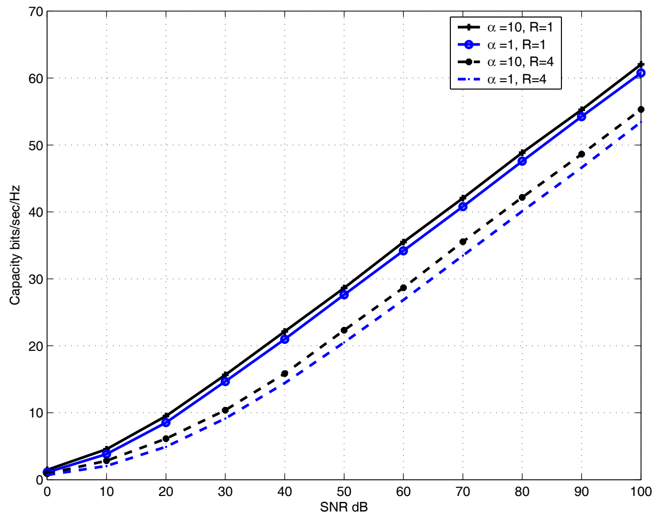

Fig. 6. Effects of amplifier gains at relay nodes for 2x2 MIMO-multiplexing relaying, $n = 1$

factor $\alpha^{2R}/\left(1+\sum_{r=1}^{R}\alpha^{2r}N^r\right)$ becomes larger as α increases. Also, the trends observed for the case $\alpha = 1$ in the previous figures are also deducible for the case $\alpha > 1$ in this figure as well.

IV. Conclusion

This paper derives the multi-hop capacity of MIMO-multiplexing relaying systems. We derive closed-form expressions for the multihop capacity of MIMO-multiplexing relaying systems, and analyze the asymptotic effect of multi-hop relaying on the MIMO capacity. We found that asymptotically, R-hops relaying degrade the capacity by at most $-N\log_2\left(\alpha^{2R}/\left(1+\sum_{r=1}^{R}\alpha^{2r}N^r\right)\right) + RN\log_2(N)$ bits/sec/Hz. Therefore, high-order MIMO systems suffer greater capacity loss due to multi-hop relaying than low-order MIMO systems. It is also observed that the capacity loss is fairly constant across all OFDM systems for a given MIMO-multiplexing configuration.

Appendix A

- *Lemma* 1: For $i = 0, \cdots, R-1$, $E[\mathbf{H}_R \cdots \mathbf{H}_{i+1}\mathbf{H}_{i+1}^H \cdots \mathbf{H}_R^H] = N^{R-i}\mathbf{I}_{nN}$.

Proof: Let $\mathbf{A}$ be a complex random square matrix satisfying $E[\mathbf{A}\mathbf{A}^H] = c\mathbf{I}_{nN}$, where c is a positive constant, and let $\mathbf{B}$ be a square matrix, with the same dimensions as $\mathbf{A}$, of block diagonal form: $\mathbf{B} = \text{diag}\{\mathbf{B}\}_{k=0}^{n-1}$, where $\mathbf{B}_0, \cdots, \mathbf{B}_{n-1}$ are mutually independent, and each $\mathbf{B}_k$ is an N x N complex random square matrix independent of $\mathbf{A}$, with iid, zero-mean entries and expected squared magnitude equal to 1. Let the rows of $\mathbf{B}$ be denoted by $\mathbf{b}_j$, $j = 1, \cdots, nN$ and the (s,t)th entry of $\mathbf{B}$ denoted by $b_{s,t}$. The (s,t)th entry of $\mathbf{B}\mathbf{A}\mathbf{A}^H\mathbf{B}^H$ is $\mathbf{b}_s\mathbf{A}\mathbf{A}^H\mathbf{b}_t^*$, where $*$ denotes complex conjugate. Clearly, $E[\mathbf{b}_s\mathbf{A}\mathbf{A}^H\mathbf{b}_t^*] = 0$ for $s \neq t$. For $s = t$, we have

$$E[\mathbf{b}_s\mathbf{A}\mathbf{A}^H\mathbf{b}_s^*] = \sum_{k=1}^{nN} E\Big[b_{s,k}\Big((\mathbf{A}\mathbf{A}^H)_{k,1}b_{s,1}^* + \cdots + (\mathbf{A}\mathbf{A}^H)_{k,nN}b_{s,nN}^*\Big)\Big].$$

In the k^{th} term above, the expectation is just $E[(\mathbf{A}\mathbf{A}^H)_{k,k}|\mathbf{b}_{s,k}|^2] = E[(\mathbf{A}\mathbf{A}^H)_{k,k}]E[|\mathbf{b}_{s,k}|^2] = cE[|b_{s,k}|^2]$. Since there are only N values of k for which $E[|b_{s,k}|^2]$ is not 0 (in which case the expectation is equal to 1), we have that $E[\mathbf{b}_s\mathbf{A}\mathbf{A}^H\mathbf{b}_s^*] = Nc$, and so $E[\mathbf{B}\mathbf{A}\mathbf{A}^H\mathbf{B}^H] = Nc\mathbf{I}_{nN}$. With $\mathbf{A} = \mathbf{H}_{i+1}$ and $\mathbf{B} = \mathbf{H}_{i+2}$, we have $E[\mathbf{H}_{i+2}\mathbf{H}_{i+1}\mathbf{H}_{i+1}^H\mathbf{H}_{i+2}^H] = N^2\mathbf{I}_{nN}$. The result follows by repeatedly applying the same argument with $\mathbf{A} = \mathbf{H}_j \cdots \mathbf{H}_{i+1}$ and $\mathbf{B} = \mathbf{H}_{j+1}$, for $j = i+2, \cdots, R-1$.

Appendix B

Eq. (20) can be expressed as

$$\begin{aligned}\mathcal{C} \approx \frac{N}{n}\sum_{k=0}^{n-1}\frac{1}{N}\sum_{q_0=0}^{N-1}\sum_{l_1^0=0}^{q_0}\sum_{l_2^0=0}^{q_0}(-1)^{l_1^0+l_2^0}A_{l_1^0}(q_0)\\ \cdot A_{l_2^0}(q_0)(l_1^0+l_2^0)!\int_0^\infty\cdots\int_0^\infty\left[\ln(a) - b + \sum_{\mu=1}^{l_1^0+l_2^0}\frac{1}{\mu}\right]\\ \cdot\prod_{j=1}^{R}\left(\frac{1}{N}\sum_{q_j=0}^{N-1}\sum_{l_1^j=0}^{q_j}\sum_{l_2^j=0}^{q_j}(-1)^{l_1^j+l_2^j}A_{l_1^j}(q_j)\right.\\ \left.\cdot A_{l_2^j}(q_j)\left(\lambda^{k,j}\right)^{l_1^j+l_2^j}e^{-\lambda^{k,j}}\right)d\lambda^{k,R}\cdots d\lambda^{k,1}. \qquad \text{(A.1)}\end{aligned}$$

Substituting $a = \rho\prod_{i=1}^{R}\lambda^{k,i}$, yields

$$\begin{aligned}\mathcal{C} \approx \frac{N}{n}\sum_{k=0}^{n-1}\frac{1}{N}\sum_{q_0=0}^{N-1}\sum_{l_1^0=0}^{q_0}\sum_{l_2^0=0}^{q_0}(-1)^{l_1^0+l_2^0}A_{l_1^0}(q_0)\\ \cdot A_{l_2^0}(q_0)(l_1^0+l_2^0)!\Big\{I_1(k)+I_2(k)+I_3(k)\Big\}, \qquad \text{(A.2)}\end{aligned}$$

where

$$\begin{aligned}I_1(k) =\int_0^\infty\cdots\int_0^\infty\ln\left(\rho\prod_{i=1}^{R}\lambda^{k,i}\right)\cdot\prod_{j=1}^{R}\left(\frac{1}{N}\sum_{q_j=0}^{N-1}\sum_{l_1^j=0}^{q_j}\sum_{l_2^j=0}^{q_j}(-1)^{l_1^j+l_2^j}\right.\\ \left.\cdot A_{l_1^j}(q_j)A_{l_2^j}(q_j)\left(\lambda^{k,j}\right)^{l_1^j+l_2^j}e^{-\lambda^{k,j}}\right)d\lambda^{k,R}\cdots d\lambda^{k,1}\end{aligned} \qquad \text{(A.3)}$$

$$\begin{aligned}I_2(k) =\int_0^\infty\cdots\int_0^\infty(-b)\cdot\prod_{j=1}^{R}\left(\frac{1}{N}\sum_{q_j=0}^{N-1}\sum_{l_1^j=0}^{q_j}\sum_{l_2^j=0}^{q_j}(-1)^{l_1^j+l_2^j}A_{l_1^j}(q_j)\right.\\ \left.\cdot A_{l_2^j}(q_j)\left(\lambda^{k,j}\right)^{l_1^j+l_2^j}e^{-\lambda^{k,j}}\right)d\lambda^{k,R}\cdots d\lambda^{k,1} \qquad \text{(A.4)}\end{aligned}$$

$$\begin{aligned}I_3(k) =\int_0^\infty\cdots\int_0^\infty\sum_{\mu=1}^{l_1^0+l_2^0}\frac{1}{\mu}\cdot\prod_{j=1}^{R}\left(\frac{1}{N}\sum_{q_j=0}^{N-1}\sum_{l_1^j=0}^{q_j}\sum_{l_2^j=0}^{q_j}(-1)^{l_1^j+l_2^j}\right.\\ \left.\cdot A_{l_1^j}(q_j)A_{l_2^j}(q_j)\left(\lambda^{k,j}\right)^{l_1^j+l_2^j}e^{-\lambda^{k,j}}\right)d\lambda^{k,R}\cdots d\lambda^{k,1}\end{aligned} \qquad \text{(A.5)}$$

Using the iid assumption of $\lambda^{k,i}$, $i = 1, \cdots, R$, we finally express these integrals as

$$I_1(k) = \ln(\rho) \cdot \prod_{j=1}^{R} \Bigg(\frac{1}{N} \sum_{q_j=0}^{N-1} \sum_{l_1^j=0}^{q_j} \sum_{l_2^j=0}^{q_j} (-1)^{l_1^j + l_2^j} A_{l_1^j}(q_j) \cdot A_{l_2^j}(q_j) \int_0^{\infty} \left(\lambda^{k,j}\right)^{l_1^j + l_2^j} e^{-\lambda^{k,j}} \, d\lambda^{k,j} \Bigg) + \sum_{i=1}^{R} \Bigg\{ \Big(\frac{1}{N} \sum_{q_i=0}^{N-1} \sum_{l_1^i=0}^{q_i} \sum_{l_2^i=0}^{q_i} (-1)^{l_1^i + l_2^i} A_{l_1^i}(q_i) \cdot A_{l_2^i}(q_i) \int_0^{\infty} \ln\left(\lambda^{k,i}\right) \left(\lambda^{k,i}\right)^{l_1^i + l_2^i} e^{-\lambda^{k,i}} \, d\lambda^{k,i} \Big) \cdot \prod_{j=1, j\neq i}^{R} \Bigg(\frac{1}{N} \sum_{q_j=0}^{N-1} \sum_{l_1^j=0}^{q_j} \sum_{l_2^j=0}^{q_j} (-1)^{l_1^j + l_2^j} A_{l_1^j}(q_j) \cdot A_{l_2^j}(q_j) \int_0^{\infty} \left(\lambda^{k,j}\right)^{l_1^j + l_2^j} e^{-\lambda^{k,j}} \, d\lambda^{k,j} \Bigg) \Bigg\}, \quad \text{(A.6)}$$

$$I_2(k) = (-b) \cdot \prod_{j=1}^{R} \Bigg(\frac{1}{N} \sum_{q_j=0}^{N-1} \sum_{l_1^j=0}^{q_j} \sum_{l_2^j=0}^{q_j} (-1)^{l_1^j + l_2^j} A_{l_1^j}(q_j) \cdot A_{l_2^j}(q_j) \int_0^{\infty} \left(\lambda^{k,j}\right)^{l_1^j + l_2^j} e^{-\lambda^{k,j}} \, d\lambda^{k,j} \Bigg), \quad \text{(A.7)}$$

and

$$I_3(k) = \sum_{\mu=1}^{l_1^0 + l_2^0} \frac{1}{\mu} \cdot \prod_{j=1}^{R} \Bigg(\frac{1}{N} \sum_{q_j=0}^{N-1} \sum_{l_1^j=0}^{q_j} \sum_{l_2^j=0}^{q_j} (-1)^{l_1^j + l_2^j} A_{l_1^j}(q_j) \cdot A_{l_2^j}(q_j) \int_0^{\infty} \left(\lambda^{k,j}\right)^{l_1^j + l_2^j} e^{-\lambda^{k,j}} \, d\lambda^{k,j} \Bigg). \quad \text{(A.8)}$$

The integrals involved in Eq. (A.6)-(A.8) have the following closed-form solutions [15]:

$$\int_0^{\infty} \lambda^d e^{-\lambda} \, d\lambda = d!, \quad \text{(A.9)}$$

$$\int_0^{\infty} \ln(\lambda) \lambda^d e^{-\lambda} \, d\lambda = \sum_{\mu=0}^{d-1} \frac{d!}{(d-\mu)} - d! b, \quad \text{(A.10)}$$

where d is a positive integer. Using Eqs. (A.9) and (A.10) in (A.6)-(A.8), and substituting Eqs. (A.6)-(A.8) in (A.2) results in Eq. (21).

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