Optimal Recruitment of Smart Vehicles for Reputation-Aware Public Sensing

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Abstract—**Public sensing services utilizing the abundant on-vehicle resources are gaining high interest nowadays. One of the challenges facing such ubiquitous utilization is the recruitment and selection of the participating vehicles. In this paper, we present an optimal reputation-aware, trajectory-based framework that handles recruitment of vehicles for public sensing. The framework considers the spatiotemporal availability of participants along with their reputation to select vehicles that achieve desired coverage of an area of interest within a budget limit. In addition, we present a reputation assessment scheme and a pricing model for computing a reputation score and a recruitment cost for each candidate participant. The framework is formulated as an integer linear programming optimization problem and hence provides a benchmark and upper bound on achievable potential. We present analysis for two different practical recruitment objectives and show results under various scenarios.**

Keywords—*Public sensing, Recruitment, Smart vehicles.*

I. INTRODUCTION

With the high benefits the public sensing paradigm brought to the service and application domain, there is interest nowadays to widen the scope of applications by engaging more resources in the sensing loop. Although smartphones have been the main players in this domain [1], their use suffers from limitations due to the scarcity of their on-board resources and their unpredictable mobility patterns. Concurrently, smart vehicles with their abundant resources are offering promising sensing solutions. With the wide variety of on-vehicle sensors, high processing and storage capabilities, and diversified communication modules, smart vehicles are becoming major enablers for remarkable public sensing-based applications [2].

In [3], we presented a categorization of the applications that can be supported by the use of vehicles as mobile sensors into instant and on-move sensing applications. In contrast to the instant sensing applications that do not require continuous readings, the on-move ones are made feasible by the mobility of vehicles and the ability to periodically report data on the go to provide on-move coverage. Examples of the former are the instantaneous reporting of weather conditions and pollution levels. Examples of the latter category include reporting of road and traffic conditions.

In the general architecture of public sensing, a service provider (SP) is responsible for collecting data from data contributors/participants. After performing required data analytics, a provider presents the sensing-based service to data consumers/end users [3].

There is an abundance of vehicles on a road with diverse capabilities. There is also a requirement for paying incentives for participating vehicles to keep them engaged. Hence effective recruitment schemes are needed to ensure the selection of the right number of participants achieving a required level of coverage for an area of interest in a cost effective manner. In a previous work [3], we introduced the trajectory-based recruitment (TBR) scheme that handles the aforementioned recruitment requirements through a heuristic greedy scheme. TBR utilizes the trajectories of vehicles as indicators of the spatiotemporal availability of potential participants.

In real implementations, with having diversity of participant's behaviour and the capabilities of participating vehicles, depending solely on availability for recruitment would not be adequate to differentiate among the variety of potential participants. With considering reputation of participants as an additional criterion, along with availability, the recruitment process would aim to maximize the benefit out of the chosen participants by selecting those who are more likely to contribute with high quality data. Furthermore, since in real scenarios SPs will have a budget cap that the total recruitment cost cannot exceed, introducing a budget constraint while selecting the participants is called for. Consequently, in this paper we present an optimal reputation-aware, trajectory-based recruitment framework that accommodates the consideration of participant's reputation and the budget constraint while building on the availability concept introduced through our TBR scheme.

The framework is formulated as an integer linear programming (ILP) optimization problem for two different recruitment objectives. The first objective considers maximizing the available coverage while minimizing the overlapping among the chosen segments to avoid data redundancy problems. The second objective targets maximizing the available coverage, while minimizing the total recruitment cost. As required inputs to the recruitment framework, a reputation score and a recruitment cost of each candidate participant need to be computed. Therefore, we introduce a reputation assessment scheme and a pricing model that feed the recruitment framework with these needed inputs. We present performance benchmarks for each recruitment objective giving SPs some performance bounds on these schemes for reputation-aware recruitment of vehicles for public sensing services.

The remainder of this paper is organized as follows. In

Section II, we discuss some related work in the areas of utilizing vehicles as a sensing resource and reputation-aware recruitment for public sensing. In Section III, we present the proposed reputation assessment scheme and pricing model. The proposed recruitment framework along with the ILP formulations are presented in Section IV. In Section V, we discuss the benchmark performance results of the two recruitment objectives. Finally, we conclude the paper and present our future work in Section VI.

II. RELATED WORK

In this section, we touch on work relevant to the areas of using vehicles as mobile sensors and reputation-aware recruitment.

Many platforms and applications are proposed that utilize on-vehicle sensors for providing public sensing-based services and applications. The MobEyes platform [4] is a popular example that focuses on using sensors in vehicles to monitor their surroundings and recognize objects, and utilize the other vehicular resources for storing the sensed data and advertising it for potential sharing upon a request. The MobEyes platform depends solely on inter-vehicular communication for its work. Many other platforms are enhanced by use of the Internet which they use for sending the sensed data to remote servers. Many examples of such platforms are discussed in [2]. Currently, there is a high focus on utilizing vehicular sensors for road condition monitoring services for the high benefits they bring. The CarMote system [5] is an example of this category of applications.

Although the aforementioned platforms succeed in utilizing the vehicular sensing resources, they lack having recruitment schemes for selecting the participating vehicles. Most of them depend on pilot vehicles for their performance evaluation purposes. For practical use, they should be coupled with recruitment modules to aid in the selection process.

In the area of reputation-aware recruitment for public sensing, a few models have been proposed in the literature that are concerned with recruitment of smartphones to utilize their on-board sensors. In [6], the authors proposed a recruitment framework that considers a participant's availability and participation habits for selection. To maximize the coverage of the area of interest within a limited budget, the authors use the greedy solution of the budgeted maximum coverage problem [7]. Their recruitment framework differs from the proposed framework in that it does not consider on-move availability as it is limited to smartphone use. In addition, it does not consider availability and reputation simultaneously; it supports selection by any of the metrics independently based on user choice.

As discussed above, the vehicular sensing platforms and reputation-aware recruitment frameworks suffer from some limitations. The work presented in this paper is stimulated by the need for handling these limitations and fulfilling the requirements of recruiting vehicles for public sensing.

III. REPUTATION ASSESSMENT AND PRICING

In this section, we present a reputation assessment scheme and a pricing model that are responsible for computing a reputation score and a recruitment cost for each participating vehicle, respectively. These parameters along with vehicles' trajectories are fed to the recruitment framework as inputs to start the selection process.

A. Reputation Assessment Scheme

Our reputation assessment scheme aims at computing a score for each candidate participant based on two main aspects: *1) Participation Commitment* and *2) Quality of Information (QoI)*. Each of these aspects can have many underlying criteria as discussed below.

1) Participation Commitment:
For assessing the participation commitment of a driver, two criteria can be considered: a) the confidence of having the participant follow the announced trajectory, and b) the willingness to participate. These two criteria can be computed based on the past history of each participant.

2) Quality of Information (QoI):
Consideration of QoI can be for the information reported by participants in previous tasks (history data) or anticipating the QoI that can be retrieved based on current participants' status. If history data will be considered, some metrics such as data accuracy, timeliness, relevance, proximity, completeness, and quality of sensed data (viz. photos or videos for a camera sensor) can be used to assist in building a participant's reputation to be considered in further recruitments. In anticipating the QoI to be retrieved, a metric such as the quality of on-vehicle resources of a participant can be considered. This can be anticipated by parameters such as a vehicle's brand, model, and manufacturing year. Such anticipated QoI can be counted on for assessing reputation of first-time participants.

A final participant reputation score r can be computed based on the normalized participation commitment p and quality of information q using an additive utility function as follows,

$$r = \alpha p + \beta q \tag{1}$$

where α and β are weights assigned based on the QoI requirements of the application such that $\alpha + \beta = 1$. Both p and q can be computed using similar utility functions that take into consideration their underlying metrics.

B. Pricing Model

Taking reputation into consideration along with participant availability, a dynamic pricing model can be adopted with participants' rewards based on their computed reputation score. Reward/price assigned to each participant is proportional to distance traversed (a measure of availability) as well. We compute a participant price pr_i which, in turn, is the cost c_i incurred by the SP for recruiting the ith participant, as follows

$$c_i = C_{\text{init}} + (C_m * d_i * r_i) \tag{2}$$

where C_{init} is a constant initial reward paid to incentivize participants, C_m is a constant cost per meter determined by the SP, d_i and r_i are the covering distance (in meters) and reputation score of participant i, respectively, for $1 \leq i \leq N$, where N is the number of potential participants.

For flexibility of implementation, the rewards and costs are represented as a number of tokens that can be mapped to any form of incentives by the SP. It is worth mentioning that the operation of the recruitment framework is generic and is not restricted to the use of the assessment scheme and pricing model presented above.

IV. THE REPUTATION-AWARE, TRAJECTORY-BASED RECRUITMENT FRAMEWORK

Navigation and positioning systems are considered main components of intelligent vehicles. In addition to providing navigational and trip guidance information to drivers, they provide input for most of vehicular applications including the safety, telematics, and diagnostics applications. We remark that with the assistance of these systems, the trajectories of vehicles can be easily acquired and utilized as a precise indicator of vehicles' availability. As mentioned earlier, we consider the participants' spatiotemporal availability as a 1st criterion for recruiting participants and achieving a required coverage. By noting that vehicles' trajectories overlap with sensing parameters (the sensing area and duration) defined in the sensing request, we can tighten our solution space to those that are spatiotemporally available in the area of interest. In addition, as trajectories represent on-move availability, they are suited for handling recruitment for the wide scope of on-move sensing applications. Fig. 1 shows an example of a trajectory segments solution space existing in the targeted area of an event.

In [3], we classified the data acquisition models into two categories; on-demand models, and unsolicited models. These two categories differ in when data is generated. In the on-demand models, data sensing and acquisition is done on-demand and upon a request from a SP. In the unsolicited models, vehicles sense their surroundings and store the sensed data without being tasked. Data holders advertise their carried data for possible interests from service providers/data collectors. It is noteworthy that the availability concept utilizing vehicles' trajectories can support the two types of data acquisition models mentioned above. In on-demand models, the trajectories considered for recruitment are those that vehicles are supposed to follow and can be obtained from the navigation software. For the unsolicited models, the trajectories are those that vehicles have already traversed and stored sensed data along.

With the diversity of drivers' behaviour and vehicles' capabilities, considering reputation of participants and their reported data is an important criterion that will aid in distinguishing among participants and picking those that ensure an adequate level of quality. As in practical implementations a SP responsible for the recruitment process will have a budget cap that cannot be exceeded, it is necessary to include a budget constraint in the selection process. Bearing in mind this perspective, we present an optimal reputation-aware, trajectory-based recruitment framework that considers both the spatiotemporal availability and reputation of participants while accommodating budget constraints in recruiting vehicles for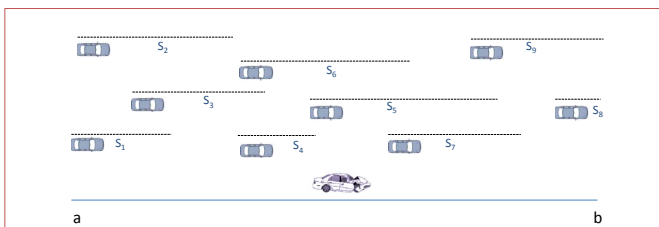 public sensing services. We aim at presenting a benchmark framework for such recruitment problems for the sake of providing the upper bounds of the recruitment solutions.

A. System Model

We consider an area of interest divided into a set of adjacent road sectors $\mathcal{T}$ of T sectors. A trajectory segment set $\mathcal{S}$ of S segments are spatiotemporally available in this area of interest. An arbitrary road sector is denoted by $k \in \mathcal{T}$ and a segment is denoted by $i \in \mathcal{S}$. Each segment $i \in \mathcal{S}$ is associated with a reputation score r_i and a recruitment cost c_i computed as in eqs. 1 & 2, respectively. A budget limit B and a reputation threshold R_{Th} will be determined by the SP interested in the recruitment process.

The system aims at finding a segment set $\mathcal{S}' \subseteq \mathcal{S}$ that achieves coverage to the sector set $\mathcal{T}$ based on a recruitment objective while considering the reputation and budget constraints, $r_i \geq R_{Th} \ \forall \ i \in \mathcal{S}'$ and $\sum_{i \in S'} c_i \leq B$, respectively.

It is worth mentioning that although Fig. 1 shows a straight road, our model is not restricted to this road topology. The proposed model is generic and can support a multiplicity of roads based on the fact that curved/non-straight roads can be treated as a series of straight roads.

Two different recruitment objectives are considered in our system that reflect practical recruitment requirements. Below, we present the recruitment problem ILP formulation for each of the recruitment objectives.

B. Problem Formulation

The recruitment problem handled by the framework is formulated as an ILP optimization problem for two different objectives. The first objective targets minimizing overlapping among the chosen segments while achieving the maximum available coverage to avoid problems resulting from data redundancy. In addition to being a waste of money, having large volumes of data redundancy unnecessarily wastes the bandwidth and overloads the transmission networks. In some other practical considerations, SPs may favor getting the covering solution with the minimum cost regardless of the level of data redundancy incurred in that solution. Therefore, we present another optimization objective that targets minimizing the total recruitment cost while achieving the maximum available coverage. Note that the solution with the minimum overlapping among segments may not correspond with the minimum cost one. The example shown in Fig. 2 demonstrates such a case.

As each of the main optimization objectives defined above involves two sub-objectives, the recruitment problem can be considered a multi-stage optimization problem. The 1st stage of the optimization formulation targets the sub-objective of maximizing the available coverage. After attaining the maximum available coverage through the 1st stage, the role of the 2nd optimization stage is refining the solution with the maximum coverage according to a 2nd sub-objective targeting either minimizing the overlapping among the chosen segments or minimizing the total recruitment cost, according to the main recruitment objective chosen by the SP.

Before discussing each stage, we introduce the following optimization variables:

- x_i : Binary decision variable set to 1 if segment i is chosen and 0 otherwise
- $\vec{x} : \vec{x} = \{x_i \in \{0, 1\} : i \in \mathcal{S}\}$
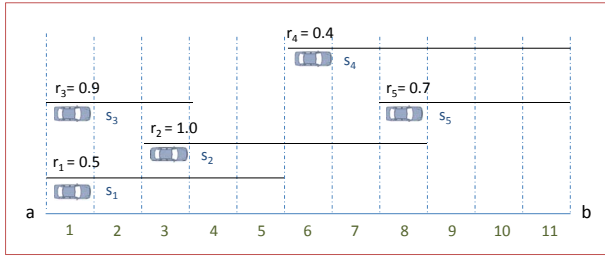

Fig. 1. An example showing trajectory segments of vehicles in an event area.

Fig. 2. An example showing that the solution with the minimum overlapping may not be the one with the minimum cost. Assume $C_{\text{init}} = 1$, $C_m = 1$, and d_i = the number of road sectors the segment covers (assuming each sector is 1m long), and c_i is computed according to Eq. 2. The solution with minimum overlapping consists of segments s_3, s_2, s_5 with a total recruitment cost = 14.5 while the solution with minimum cost consists of segments s_1, s_2, s_4 with a total recruitment cost = 13.9.

- **A** : Matrix of size TxS representing the mapping of the S segments to the T sectors
- t_k : Number of selected trajectory segments covering sector k
- $\vec{t}$: $\vec{t} = \{t_k : k \in \mathcal{T}\}$ and is computed as $\vec{t} = \mathbf{A}\vec{x}$.
- t'_k : Binary variable set to 1 if sector k is covered and 0 otherwise and is computed as $\min(t_k, 1)$
- l_i : Length of trajectory segment i

To illustrate how the mapping matrix **A** is formed and $\vec{t}$ is computed, Fig. 3 shows an example of the computation $\vec{t} = \mathbf{A}\vec{x}$ to the minimum cost solution of the use case in Fig. 2. Note that the rows of **A** represent the sectors and the columns represent the segments.

As the 1st optimization stage targeting the maximum coverage is common to the two main recruitment objectives, we present its ILP formulation below then we present the 2nd stage of the two main objectives separately in sub-sections following.

The 1st stage of optimization targeting the maximum coverage:

$$\text{Maximize} \quad \frac{\sum_{k=1}^{T} t'_k}{T} \tag{3}$$

subject to

$$C1: \quad r_i x_i \geq R_{Th} x_i \qquad \forall\, i \in S$$

$$C2: \quad \sum_{i=1}^{S} c_i x_i \leq B$$

$$C3: \quad \vec{t} - \mathbf{A}\vec{x} = \vec{0}$$

$$C4: \quad t'_k \leq 1 \qquad \forall\, k \in T$$

$$C5: \quad t'_k \leq t_k \qquad \forall\, k \in T$$

Eq. 3 is the objective function maximizing the ratio of the covered sectors. C1 and C2 are the reputation and budget constraints, respectively. C3 ensures that the vector $\vec{t}$ holds the number of selected segments covering each road sector k through a multiplication of the vector $\vec{x}$ and matrix **A** as defined earlier. C4 and C5 map the value of t_k into a binary value stored in t'_k for each sector k, and they represent the equation $t'_k = min(t_k, 1)$.

The coverage ratio of the solution obtained in this stage represents the maximum coverage, $V_{\max}$, that can be achieved by the set of available trajectory segments (vehicles).

Fig. 3. An example illustrating the computation $\vec{t} = \mathbf{A}\vec{x}$. The values relate to the minimum cost solution obtained in the use case of Fig. 2.

1) Maximum Coverage with Minimum Overlapping:
As the solution of the 1st stage may involve overlapping among the chosen segments for the sake of maximizing coverage, we consider a 2nd optimization stage with a targeted objective of minimizing segments' overlapping, while achieving the maximum coverage bound obtained from the 1st stage. The formulation of this stage is presented below.

The 2nd stage of optimization targeting the minimum overlapping:

$$\text{Minimize} \sum_{i=1}^{S} l_i x_i \tag{4}$$

subject to

$$C1, C2, C3, C4, C5$$

$$C6: \quad \frac{\sum_{k=1}^{T} t'_k}{T} \geq V_{\max}$$

Eq. 4 is the objective function minimizing the overlapping among the chosen segments through minimizing the sum of the chosen segments' length. C6 ensures that the solution obtained guarantees the maximum coverage ratio obtained from the 1st stage.

2) Maximum Coverage with Minimum Cost:
The 1st optimization stage may find many solutions achieving the maximum coverage but each with a different recruitment cost. To handle recruitment with the minimum cost desire, we consider a 2nd optimization stage targeting minimizing the total recruitment cost while achieving the maximum coverage bound obtained from the 1st stage. The formulation of this stage is presented below.

The 2nd stage of optimization targeting the minimum cost:

$$\text{Minimize} \sum_{i=1}^{S} c_i x_i \tag{5}$$

subject to

$$C1, C3, C4, C5$$

$$C7: \quad \frac{\sum_{k=1}^{T} t'_k}{T} \geq V_{\text{des}}$$

where V_{des} is the desired coverage ratio determined by the SP with its upper bound being the possible maximum coverage ratio, $V_{\max}$, obtained from the 1st stage.
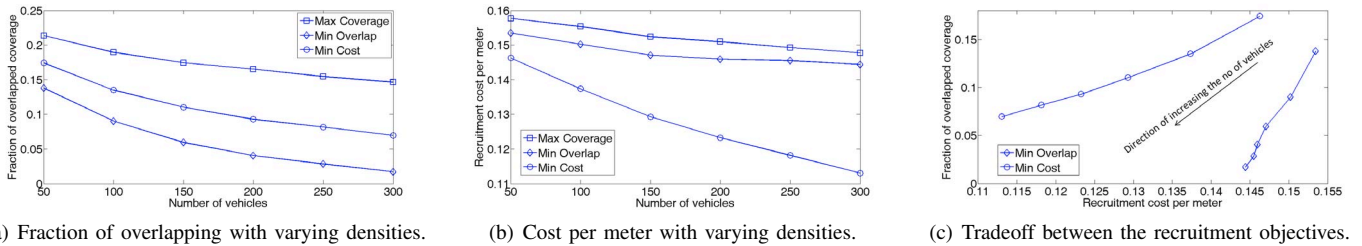
(a) Fraction of overlapping with varying densities.

(b) Cost per meter with varying densities.

(c) Tradeoff between the recruitment objectives.

Fig. 4. Performance results with B = 1000 and full coverage achieved.

(a) Fraction of overlapping with varying densities.

(b) Cost per meter with varying densities.
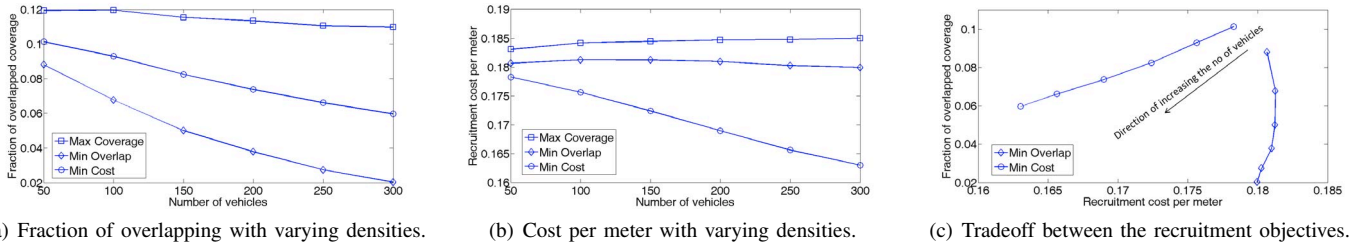
(c) Tradeoff between the recruitment objectives.

Fig. 5. Performance results with B = 650 and maximum (not full) coverage achieved.

Eq. 5 is the objective function minimizing the total recruitment cost. C7 ensures that the solution obtained guarantees the desired coverage ratio.

V. PERFORMANCE EVALUATION

In this section, we present numerical results of the proposed recruitment framework with its two recruitment objectives. The results represent upper bounds of the budgeted reputation-aware recruitment that can be achieved. We compare the solutions of the two main objectives: 1) maximum coverage with minimum overlapping (which we refer to as Min Overlap) and 2) maximum coverage with minimum cost (which we refer to as Min Cost). In addition, to show the gain achieved through the 2nd stage of optimization, we compare the solutions to that obtained from the 1st stage targeting the maximum coverage only (which we refer to as Max Coverage). The three solutions are compared in terms of the fraction of the overlapped coverage to the length of the total achieved coverage and the recruitment cost per meter.

A. Simulation Setup

We use Gurobi 5.1 [8] to solve the ILP optimization problems with Matlab as a simulation environment. We simulate an area of interest of $5Km$ divided into 100 road sectors each is 50 meters long. C_{init} is set to 1, C_m is set to 0.01, R_{Th} is set to 0.4, r_i is assigned a random value in the interval $[0,1]$, and c_i is computed according to Eq. 2 $\forall~i \in S$.

B. Numerical Results and Analysis

First, we compare the three solutions in terms of the aforementioned metrics with a budget limit that allows for achieving full coverage to the area of interest (B is set to 1000). Fig. 4 shows the results of this comparison for various densities of vehicles (number of vehicles per sector). In terms of the fraction of overlapping, as expected the Min Overlap solution gives the best results as shown in Fig. 4(a). Min Cost

solution improves on pure Max Coverage since while it is trying to minimize the cost, it may avoid segments with very long overlapping as opposed to the Max Coverage one that does not consider either the cost or overlapping. In Fig. 4(b), the solutions are compared in terms of the recruitment cost per meter with the Min Cost solution achieving the best performance. Min Overlap works better than Max Coverage in terms of cost because minimizing overlapping may implicitly lead to reducing the cost. In Fig. 4(c), we plot the performance of the Min Overlap and Min Cost together with various densities in terms of the two performance metrics for the sake of showing the tradeoff between minimizing the overlapping and minimizing the cost.

Second, we perform the same comparison but with a budget limit that may not allow for achieving full coverage (B is set to 650). In this case solutions will work on achieving the maximum available coverage. Results are shown in Fig. 5. The Min Overlap works the best in terms of the fraction of the overlapped coverage as shown in Fig. 5(a). The effect of having the limited budget can be seen in the lower improvements achieved while increasing the number of segments compared to the improvements shown in Fig. 4(a). The reason is that even with increasing the selection options by increasing the number of vehicles, many of these options cannot be considered due to the limited budget that restricts the selections process. In terms of recruitment cost, we can see in Fig. 5(b) that the Min Cost solution is the only one that improves with increasing the number of vehicles as it gets more options towards achieving its objective. The Min Overlap and Max Coverage solutions keeps almost the same results even with increasing the selection options as they both try to achieve their objectives limited by the budget cap so their recruitment cost will always be close to the limited budget cap. In Fig. 5(c), we show the tradeoff between the main objectives as in Fig. 4(c).

Finally, in Fig. 6, we study the effect of changing the reputation threshold R_{Th} while keeping the density of vehicles fixed (150 vehicles per the 5Km area). We compare the performance of the three solutions with different values of R_{Th}
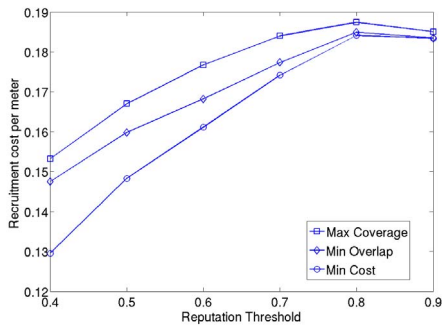
Fig. 6. Cost per meter with varying reputation thresholds.

in terms of the recruitment cost per meter metric. We can see that with increasing R_{Th}, the three solutions converge because they all will be restricted to very limited options which are the vehicles with r_i above the threshold.

VI. Conclusions and Future Work

In this paper, we proposed an optimal reputation-aware, trajectory-based framework for recruiting vehicles for public sensing services. The framework utilizes the spatiotemporal availability of participants and their reputation to find a set of vehicles that achieves coverage of an area of interest with a limited budget. We also proposed a reputation assessment scheme and a pricing model that are used to feed the framework with a reputation score and a recruitment cost of each candidate participant. We formulated the framework as an integer linear programming (ILP) optimization problem for two different recruitment objectives; maximizing coverage with minimum overlapping and maximizing coverage with minimum cost. We presented the optimal numerical results of the two recruitment objectives providing performance benchmarks and upper bounds of the recruitment solutions.

Our future work includes devising greedy heuristic algorithms that can provide near-optimal solutions to the problem studied in this paper. Such heuristic solutions are needed to handle real-time services. In addition, more techniques will be investigated to provide reputation assessment in both short and long terms.

Acknowledgement

References

[1] S. Kanhere, "Participatory sensing: Crowdsourcing data from mobile smartphones in urban spaces," in *Distributed Computing and Internet Technology*, ser. Lecture Notes in Computer Science, C. Hota and P. Srimani, Eds. Springer Berlin Heidelberg, 2013, vol. 7753, pp. 19–26.

[2] U. Lee and M. Gerla, "A survey of urban vehicular sensing platforms," *Computer Networks*, vol. 54, no. 4, pp. 527–544, 2010.

[3] S. A. Hamid, G. Takahara, and H. S. Hassanein, "On the recruitment of smart vehicles for urban sensing," in *IEEE Global Communications Conference (GLOBECOM '13)*, 2013, pp. 58–63.

[4] U. Lee, E. Magistretti, M. Gerla, P. Bellavista, and A. Corradi, "Dissemination and harvesting of urban data using vehicular sensing platforms," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 2, pp. 882–901, 2009.

[5] A. Mednis, A. Elsts, and L. Selavo, "Embedded solution for road condition monitoring using vehicular sensor networks," in *the 6th IEEE International Conference on Application of Information and Communication Technologies (AICT'12)*, 2012, pp. 1–5.

[6] S. Reddy, D. Estrin, and M. Srivastava, "Recruitment framework for participatory sensing data collections," in *the 8th International Conference on Pervasive Computing, vol. 6030 of Lecture Notes in Computer Science, Springer-Verlag*, 2010, pp. 138–155.

[7] S. Khullera, A. Mossb, and J. Naor, "The budgeted maximum coverage problem," *Information Processing Letters*, vol. 70, pp. 39–45, 1999.

[8] "Gurobi optimization," http://www.gurobi.com/, accessed: 2013-10-10.