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Published in IET Communications
Received on 7th November 2013
Revised on 17th January 2014
Accepted on 29th January 2014
doi: 10.1049/iet-com.2013.1000

ISSN 1751-8628

# Performance analysis of relay-multiplexing scheme in cellular systems employing massive multiple-input multiple-output antennas

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**Abstract:** This study presents the symbol error probability (SEP) analysis of relay-multiplexing scheme in relay-assisted cellular systems such as the IEEE 802.16j and upcoming fifth generation (5G) systems. The availability of multiple relay paths in these systems motivates two relay configurations: (i) relay diversity and (ii) relay multiplexing. Available works in the literature have focused on the relay diversity methods. In this study, we explore the relay-multiplexing alternative whereby relay stations (RSs) act as independent data pipes for transmitting multiple independent data streams from the base station (BS) to the mobile station (MS). We examine the case when the BS is equipped with massive multiple-input multiple-output (MIMO) antennas, transmitting independent data streams to several MS simultaneously via different relay paths. We derive analytical expressions for the SEP performance of the proposed scheme, and also complement the analysis with simulations. The results show that parallel relaying of independent data streams via different RSs has acceptable SEP performance while boosting the capacity of the system linearly with the number of parallel RSs available in the system. The results also show that when this relaying approach is combined with the emerging massive MIMO techniques, a tremendous boost in data rate can be achieved.

## 1 Introduction

Fifth generation (5G) cellular systems are required to provide gigabit wireless data rate, supporting bandwidth-intensive applications such as streaming and cloud computing, on mobile devices. To meet these requirements, novel capacity enhancement techniques are currently being explored for the 5G cellular systems. In the current fourth generation (4G) cellular standards such as the IEEE802.16j and the long-term evolution (LTE), multihop relaying has been included as an integral part of network design. It can be used to achieve capacity or throughput enhancements whenever necessary, via the use of relay stations (RSs) inserted in the transmission path between the base station (BS) and the mobile station (MS). The transmission of data from the BS to the MS is thus conducted in two hops in such cases; first hop features data transfer from the BS to the RSs, whereas the second hop features data relaying from the RS to the MS. Since the introduction of multiple-input multiple-output (MIMO) systems, many research works have demonstrated various benefits achievable when MIMO systems are deployed in different scenarios. Recently, the use of aggressively large number of antennas at the BS (or massive MIMO system) has gained attentions from system developers as a possible way forward for realising gigabit data rate in the future 5G systems. In this work, we consider massive MIMO multiplexing system. Among the detection schemes available for use in MIMO multiplexing systems, maximum likelihood detection (MLD) provides the optimal performance. However the exact symbol error probability (SEP) analysis of MIMO multiplexing with maximum likelihood (ML) detection was not available in the literature until 2009 when the authors in [1] provided a very accurate SEP analysis for arbitrary MIMO multiplexing systems. Here we extend the analytical approach in [1] to study the performance of the proposed relay-multiplexing scheme, in a massive multi-user MIMO context. MIMO multiplexing is well known to be a major enabler for high-speed wireless transmission. Relay-assisted systems are also known to provide data rate enhancements compared to an equivalent one-hop transmission without relaying. A combination of these duo in a massive multiuser MIMO (MU-MIMO) context, as explored in this paper, is therefore interesting and somewhat indispensable in broadband cellular networks.

Two-hop and multihop relaying have been investigated in many works [2–9]. Most of these works, however, considered the relay diversity approach, where the relayed data from all the RSs at any time instant belong to the same data sub-stream which may be coded or uncoded. A comprehensive survey of these relaying transmission approaches was presented recently in [9], for both the time-division duplexing and frequency-division duplexing systems. The paper also discussed MU-MIMO system using space-division multiple access. Similar to all other existing works however, the work in [9] focuses on the use of RSs in cooperative (diversity) transmission modes. In [10, 11] the authors provide initial results on the relay-multiplexing alternative, whereby multiple RSs are employed for parallel relaying of independent data streams in an IEEE 802.16j setup. In the IEEE 802.16j relaying configuration, data transfer from the BS to the MS is conducted in two phases. The first phase involves data transmission from the BS to the RSs, whereas the second phase involves data relaying from the RSs to the MS as shown in Fig. 1. Similar relaying configuration has been supported recently in the 4G-LTE system, and extensive use of relays is expected in the upcoming 5G systems. This is because system developers will most likely adopt 5G deployments in the mm-wave bands, where BS transmissions must be aided by relays owing to the limited range and penetration abilities of mm-wave signals [12].

In this paper, we provide the SEP analysis of relay-multiplexing scheme for cellular networks such as the IEEE 802.16j, the 4G-LTE and the upcoming 5G systems, where massive MIMO antennas are deployed at the BS. In our analysis, we apply transmit preprocessing schemes at the BS to partition the channel between the BS and the RSs into K independent parallel relay paths with no interference among them. Thus, the transmission of the independent data streams from the BS to the RSs in the first hop can be treated as a massive MU-MIMO downlink system when large numbers of antennas are employed. For the transmission of data from the RS to the MS in the second hop, we consider that the RSs may service one end user, corresponding to the case where relay multiplexing and massive MIMO antennas are employed for enhancing the data rate of an individual user, or that the RSs may service multiple independent end users, corresponding to the case where relay multiplexing and massive MIMO antennas are employed for increasing the number of users supported in the network. We show that this case still has acceptable SEP performance, while providing interesting capacity boost in the network.

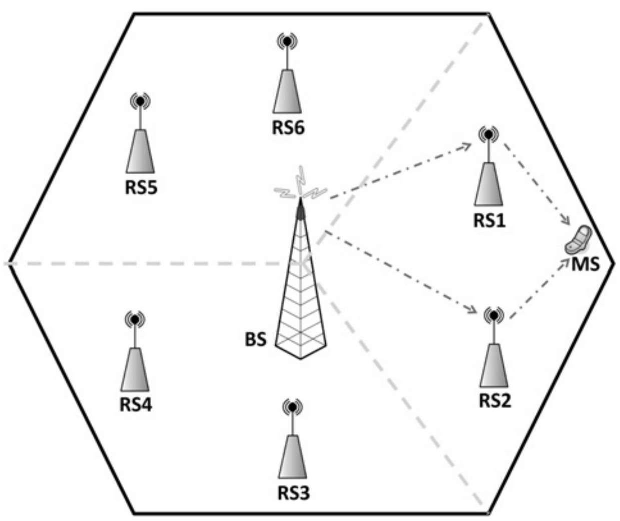


**Fig. 1** *Relay-assisted cellular network (IEEE 802.16j configuration)*

### 1.1 Contribution of paper

The contribution of this paper includes the extension of the existing MIMO-MLD analysis to relay-based transmission, and the SEP analysis of massive MU-MIMO systems with relaying. To the best of the authors' knowledge, these works have not been previously presented in the literature.

The rest of the paper is organised as follows. Section 2 presents the performance analysis of MIMO multiplexing systems for single- and two-hop configurations employing MLD at the receiver. In Section 3, we provide the SEP analysis for relay-multiplexing systems employing null-space-based MU-MIMO precoding. Section 4 presents the SEP analysis for relay-multiplexing systems employing SVD-assisted MU-MIMO precoding. Capacity enhancements offered by relay-multiplexing employed with massive MIMO systems are discussed in Section 5. Simulation results are discussed in Section 6, while Section 7 presents the conclusion of the paper.

## 2 Performance analysis of MIMO-MLD scheme with and without relaying

This section first summarises an existing result in the literature [1] for the SEP of one-hop MIMO multiplexing system, and then extends it to two-hop MIMO multiplexing systems.

### 2.1 One-hop MIMO multiplexing system

This subsection refers to the configuration shown in Fig. 2*a* where the BS is equipped with $N_{\rm t,BS}$ transmit antennas and the MS is equipped with $N_{\rm r,MS}$ receive antennas resulting in an $N_{\rm t,BS} \times N_{\rm r,MS}$ MIMO multiplexing system. $\boldsymbol{x} = [x_1, x_2, \ldots, x_{N_{\rm t,BS}}]^{\rm T}$ is the data symbol vector transmitted by the BS towards the MS. The signal vector received at the MS is given by

$$\boldsymbol{y} = \boldsymbol{Hx} + \boldsymbol{n} \qquad (1)$$

where $\boldsymbol{y} = [y_1, \ldots, y_{N_{\rm r,MS}}]^{\rm T}$ and $y_j$ denotes the signal received at the $j$th MS antenna. Each element of the vector $\boldsymbol{n} = [n_1, \ldots, n_{\rm r,MS}]^{\rm T}$ has additive white Gaussian noise (AWGN) distribution with zero mean and variance $\sigma_n^2$. $\boldsymbol{H}$ represents the channel matrix with each element being i.i.d. complex Gaussian variable with zero mean and unit variance. If all the symbols in the constellation are transmitted with equal probability then the ML detection can be written as

$$\tilde{x} = \underset{x \in \mathcal{X}}{\operatorname{argmin}} \sum_{j=1}^{N_{\rm r,MS}} \left| y_j - \sum_{i=1}^{N_{\rm t,BS}} h_{j,i} x_i \right|^2 \qquad (2)$$

where $\mathcal{X}$ represents the constellation size of the modulation scheme used and $\tilde{\boldsymbol{x}}$ denotes the decision vector for $\boldsymbol{x}$. Recently, for the first time in the literature, Peng *et al.* [1] provided an accurate result for the SEP of MIMO-MLD system. A summary of their analytical approach is discussed in the following. Let $x_i$ denotes the symbol transmitted from the $i$th antenna and $\bar{\boldsymbol{x}}_i = \{x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_{N_{\rm t,BS}}\}$ presents the symbols transmitted from all other antennas of the BS. The SEP of $x_i$ can be expressed as a combination of SEPs conditioned on a set of error events identical to $\bar{\boldsymbol{x}}_i$ and their probabilities [1, Eq.


doi: 10.1049/iet-com.2013.1000

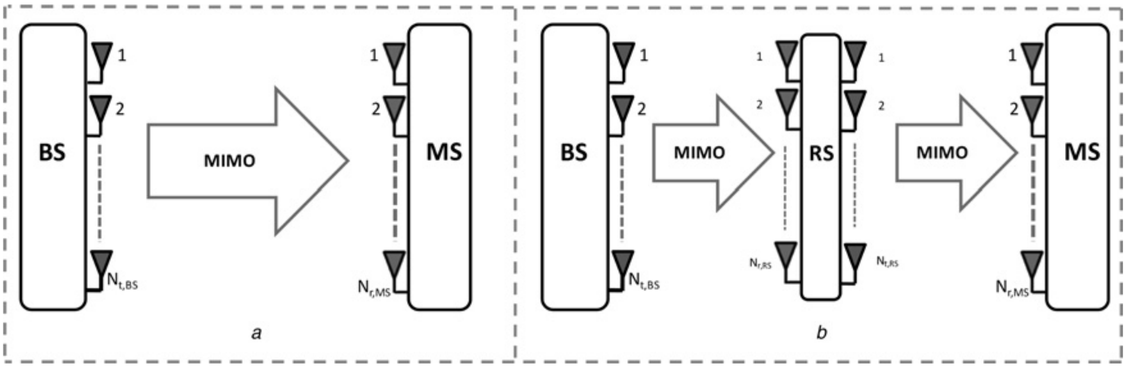


**Fig. 2** *Multiplexing system*
*a* One-hop MIMO
*b* Two-hop MIMO

(5)] as

$$P(\tilde{x}_i \neq x_i) = P(\tilde{x}_i \neq x_i|\mathrm{err}(\phi))P(\mathrm{err}(\phi)) + \sum_{k=1,k\neq i}^{N_{\mathrm{t,BS}}} P(\tilde{x}_i \neq x_i|\mathrm{err}(k))P(\mathrm{err}(k)) + \sum_{l=1,l\neq i}^{N_{\mathrm{t,BS}}} \sum_{m=l+1,m\neq i}^{N_{\mathrm{t,BS}}} P(\tilde{x}_i \neq x_i|\mathrm{err}(l,m))P(\mathrm{err}(l,m)) \cdots + P(\tilde{x}_i \neq x_i|\mathrm{err}(\Omega))P(\mathrm{err}(\Omega)) \quad (3)$$

where err($\phi$), err($k$), err($l$, $m$) and err($\Omega$) correspond to no error, one error ($\tilde{x}_k \neq x_k$), two errors ($\tilde{x}_l \neq x_l$, $\tilde{x}_m \neq x_m$) and all errors, respectively, in $\bar{\boldsymbol{x}}_i$. Suppose err(·) denotes an error event in $\bar{\boldsymbol{x}}_i$, then $P(\tilde{x}_i \neq x_i|\mathrm{err}(\cdot))$ presents the SEP of $x_i$ conditioned on err(·) and $P(\mathrm{err}(\cdot))$ denotes the probability of an error event [1]. When no error occurs in transmitted symbols $\bar{\boldsymbol{x}}_i$ that is the event err($\phi$), the detection of $x_i$ can be written as

$$\tilde{x}_i = \underset{x_i\in\mathcal{X}}{\operatorname{argmin}} \sum_{j=1}^{N_{\mathrm{r,MS}}} |h_{j,i}x_i + n_j - h_{j,i}x_i| \quad (4)$$

The post-detection signal-to-noise ratio (SNR) is given as

$$\gamma_{i,\mathrm{err}(\phi)} = \sum_{j=1}^{N_{\mathrm{r,MS}}} \gamma_{j,i,\mathrm{err}(\phi)} = \frac{\sum_{j=1}^{N_{\mathrm{r,MS}}} |h_{j,i}|^2|x_i|^2}{\sigma_n^2} = \frac{\omega_i|x_i|^2}{\sigma_n^2} \quad (5)$$

where $\omega_i = \sum_{j=1}^{N_{\mathrm{r,MS}}} |h_{j,i}|^2$ and $\gamma_{j,\ i,\mathrm{err}(\phi)}$ represents SNR at the $j$th receive antenna from the $i$th transmit antenna. For 4QAM modulation, the SEP conditioned on $x_i$, $\omega_i$ and err($\phi$), that is no error in $\bar{\boldsymbol{x}}_i$, can be written as

$$p(\tilde{x}_i \neq x_i|x_i, \omega_i, \mathrm{err}(\phi)) = 2Q\left(\sqrt{\gamma_{i,\mathrm{err}(\phi)}}\right) - Q^2\left(\sqrt{\gamma_{i,\mathrm{err}(\phi)}}\right) \quad (6)$$

By using the statistics of $x_i$ and $\omega_i$ in (6), the conditional SEP can be expressed as [1]

$$P(\tilde{x}_i \neq x_i|\mathrm{err}(\phi)) = \frac{1}{\pi}\sum_{x_i\in\mathcal{X}} p(x_i)\left[\frac{3\pi}{4} + \sum_{i=1}^{N_{\mathrm{r,MS}}}(-1)^i \sum_{j=0}^{i-1} \frac{\alpha_{\mathrm{err}(\phi)}^{i-j-1/2}}{(1+\alpha_{\mathrm{err}(\phi)})^{i-1/2}} \times \binom{N_{\mathrm{r,MS}}}{i}\binom{i-1}{j} I\left(0,\ -\sqrt{\frac{1+\alpha_{\mathrm{err}(\phi)}}{\alpha_{\mathrm{err}(\phi)}}},\ (j+1)\right)\right] \quad (7)$$

where $\alpha_{\mathrm{err}(\phi)} = |x_i|^2/2\sigma_n^2$ and $I(\cdot,\cdot,\cdot)$ is defined in [1, Eq. (12)] as

$$I(t_1, t_2, n) = \frac{(2n-3)!!}{2^{n-1}(n-1)!}(\arctan(t_2) - \arctan(t_1)) + \frac{(2n-3)!!}{(n-1)!}\sum_{k=1}^{n-1}\frac{(n-k-1)!}{2^k(2n-2k-1)!!} \cdot \left(\frac{t_2}{(1+t_2^2)^{n-k}} - \frac{t_1}{(1+t_1^2)^{n-k}}\right)$$

where (·)!! represents the double factorial. Similarly the SEP conditioned on one error event in the other transmitted symbols, that is one error in $\bar{\boldsymbol{x}}_i$, can also be formulated in a similar manner as [1]

$$P(\tilde{x}_i \neq x_i|\mathrm{err}(k)) = \frac{1}{\pi}\sum_{x_i\in\mathcal{X}} p(x_i)\left[\frac{3\pi}{4} + \sum_{i=1}^{N_{\mathrm{r,MS}}}(-1)^i \sum_{j=0}^{i-1} \frac{\alpha_{\mathrm{err}(k)}^{i-j-1/2}}{(1+\alpha_{\mathrm{err}(k)})^{i-1/2}} \times \binom{N_{\mathrm{r,MS}}}{i}\binom{i-1}{j} I\left(0,\ -\sqrt{\frac{1+\alpha_{\mathrm{err}(k)}}{\alpha_{\mathrm{err}(k)}}},\ (j+1)\right)\right] \quad (8)$$

where $\alpha_{\mathrm{err}(k)} = |x_i|^2/2\sigma_k^2$ and $\sigma_k^2 = \mathrm{var}\{h_{j,k}\Delta x_k + n_j\} = \sigma_{h_{j,k}}^2|\Delta x|^2 + \sigma_n^2 \simeq \alpha_k E|x_k|^2 + \sigma_n^2$. Following the same procedure as above, the SEP conditioned on two or more error events can also be derived. Finally the average SEP conditioned on all error events, that is when all symbols are



doi: 10.1049/iet-com.2013.1000

in error in $\bar{\boldsymbol{x}}_i$, can be derived as [1]

$$P(\tilde{x}_i \neq x_i|\text{err}(\Omega)) = \frac{1}{\pi}\sum_{x_i\in\mathcal{X}} p(x_i)\left[\frac{3\pi}{4} + \sum_{i=1}^{N_{\text{r,MS}}}(-1)^i \sum_{j=0}^{i-1} \frac{\alpha_{\text{err}(\Omega)}^{i-j-1/2}}{(1+\alpha_{\text{err}(\Omega)})^{i-1/2}} \times \binom{N_{\text{r,MS}}}{i}\binom{i-1}{j} I\left(0, -\sqrt{\frac{1+\alpha_{\text{err}(\Omega)}}{\alpha_{\text{err}(\Omega)}}}, (j+1)\right)\right] \quad (9)$$

where $\alpha_{\text{err}(\Omega)} = |x_i|^2/2\sigma_\Omega^2$ and $\sigma_\Omega^2 = \text{var}\{\sum_{k=1,k\neq i}^{N_{\text{t,BS}}} h_{j,k}\Delta x_k + n_j\} \simeq \sum_{k=1,k\neq i}^{N_{\text{t,BS}}} \alpha_k E|x_k|^2 + \sigma_n^2$. The probabilities $P[\text{err}(\phi)]$, $P[\text{err}(k)]$, $P[\text{err}(l, m), \ldots, P[\text{err}(\Omega)]]$ are given as [1]

$$P(\text{err}(\phi)) = \prod_{j=1,j\neq i}^{N_{\text{t,BS}}} (1 - P(\tilde{x}_j \neq x_j))$$

$$P(\text{err}(k)) = P(\tilde{x}_k \neq x_k) \prod_{j=1,j\neq i,k}^{N_{\text{t,BS}}} (1 - P(\tilde{x}_j \neq x_j))$$

$$P(\text{err}(l,m)) = P(\tilde{x}_l \neq x_l)P(\tilde{x}_m \neq x_m) \prod_{j=1,j\neq i,l,m}^{N_{\text{t,BS}}} (1 - P(\tilde{x}_j \neq x_j))$$

$$\vdots$$

$$P(\text{err}(\Omega)) = \prod_{j=1,j\neq i}^{N_{\text{t,BS}}} P(\tilde{x}_j \neq x_j) \quad (10)$$

When power is equally divided among all the transmitted symbols, the SEP of each transmitted symbol remains the same and is expressed as [1]

$$P(\text{err}) = \sum_{j=0}^{N_{\text{t,BS}}-1} \binom{N_{\text{t,BS}}-1}{j} \beta_j P(\text{err})^j (1 - P(\text{err}))^{N_{\text{t,BS}}-j-1} \quad (11)$$

where $\beta_0 = P(\tilde{x}_i \neq x_i|\text{err}(\phi))$, $\beta_j = P(\tilde{x}_i \neq x_i|\text{err}(l_1, \ldots, l_j))$ and $(l_1, \ldots, l_j)$ denotes a $j$-element subset of $\{1, \ldots, i-1, i+1, \ldots, N_{\text{t,BS}}\}$. The SEP $P(\text{err})$ can then be obtained by solving (11) for $P(\text{err})$ [1].

### 2.2 Two-hop MIMO multiplexing system

In this subsection we extend the SEP analysis for MIMO MLD presented in the previous subsection, to the two-hop MIMO with MLD at the RS and MS. Our objective is to use DF protocol in order to examine the best SEP performance possible using relaying to enhance the system data rate. Consider a MIMO relaying system, as shown in Fig. 2*b*, where the BS with $N_{\text{t,BS}}$ transmit antennas is communicating with an MS carrying $N_{\text{r,MS}}$ receive antennas via a single RS carrying $N_{\text{r,RS}}$ receive and $N_{\text{t,RS}}$ transmit antennas. $\boldsymbol{H}$ and $\boldsymbol{G}$ denote the first- and second-hop channel matrices, respectively. The $(j, i)$th element of these matrices is assumed to be an i.i.d. complex Gaussian variable with zero mean and unit variance ($\sigma_{h_{j,i}}^2 = \sigma_{g_{j,i}}^2 = 1$). As DF protocol is assumed at the RS, the post-detection SNR at the RS and MS conditioned on no error err($\phi$), one error err($k$), two errors err($l$, $m$) and all errors err($\Omega$) in the set of other transmitted symbols can be found in a similar manner as presented in the previous sub-section. Conditional SEPs can then be computed based on these SNR expressions. Finally, the average SEPs of the first- and second-hop transmissions can be written, respectively, as

$$P(\text{err})_{\text{hop}_1} = \sum_{j=0}^{N_{\text{t,BS}}-1} \binom{N_{\text{t,BS}}-1}{j} \times \beta_j P(\text{err})_{\text{hop}_1}^j (1 - P(\text{err})_{\text{hop}_1})^{N_{\text{t,BS}}-j-1} \quad (12)$$

$$P(\text{err})_{\text{hop}_2} = \sum_{j=0}^{N_{\text{t,RS}}-1} \binom{N_{\text{t,RS}}-1}{j} \times \beta_j P(\text{err})_{\text{hop}_2}^j (1 - P(\text{err})_{\text{hop}_2})^{N_{\text{t,RS}}-j-1} \quad (13)$$

As the transmitted symbols are decoded twice in cascade, the overall SEP of two-hop MIMO relaying system is given by [13]

$$P(\text{err}) = P(\text{err})_{\text{hop}_1} + P(\text{err})_{\text{hop}_2} - P(\text{err})_{\text{hop}_1} P(\text{err})_{\text{hop}_2} \quad (14)$$

where $P(\text{err})_{\text{hop}_i}$ denotes the average SEP of the $i$th ($i = 1, 2$) hop. Next, we derive the closed-form expressions for $P(\text{err})_{\text{hop}_1}$ and $P(\text{err})_{\text{hop}_2}$ for relay-multiplexing systems. To the best of authors' knowledge, these expressions are new contributions. In our derivations, we consider two popular approaches available in the literature that can be employed for the multiuser access problem in the proposed relay-multiplexing scheme, when transmitting parallel data streams from the BS to MS via multiple RSs. These approaches are presented in Sections 3 and 4.

## 3 Relay-multiplexing system with null-space-based MU-MIMO precoding

The first approach, introduced in [14], involves using the null-space-based MU-MIMO channel decomposition at the transmitter such that when sending data targeted to a particular RS, the precoding matrix employed ensures that the undesired RSs are completely nulled, removing the multiple relay interference.

Next, we present the SEP analysis of massive MU-MIMO system employing relay-multiplexing scheme as shown in Fig. 3. Consider that the relay-assisted cellular system shown in Fig. 1 assumes the configuration shown in Fig. 3, where a BS equipped with $N_{\text{t,BS}}$ antennas communicates simultaneously with several MSs, each equipped with $N_{\text{r,MS}_l}$ antennas, via $K$ multi-antenna RSs with each RS servicing $L$ MSs at a time. It is assumed that $\text{RS}_j$ has $N_{\text{r,RS}_j}$ receiving and $N_{\text{t,RS}_j}$ transmitting antennas such that $N_{\text{t,BS}} = \sum_{j=1}^{K} N_{\text{r,RS}_j} = \sum_{j=1}^{K} N_{\text{t,RS}_j} = \sum_{j=1}^{K}\sum_{l=1}^{L} N_{\text{r,MS}_{j,l}}$ and that DF protocol is employed at all the $K$ RSs. The BS serves up to $K \times L$ MSs. The channel between BS and $K$ RSs is divided into parallel MU-MIMO channels, using null-space-based precoding at the BS and post-processing at $\text{RS}_j$. The parameter $\boldsymbol{H}_j$ denotes the channel matrix from the BS to $\text{RS}_j$, whereas $\boldsymbol{G}_{l,j}$ denotes the channel matrix from $\text{RS}_j$ to the $l$th MS. We assume that the BS and $\text{RS}_j$ have accurate knowledge of their forward channels. The BS broadcasts $K$ independent data streams $z_1, z_2, \ldots, z_K$ towards


doi: 10.1049/iet-com.2013.1000

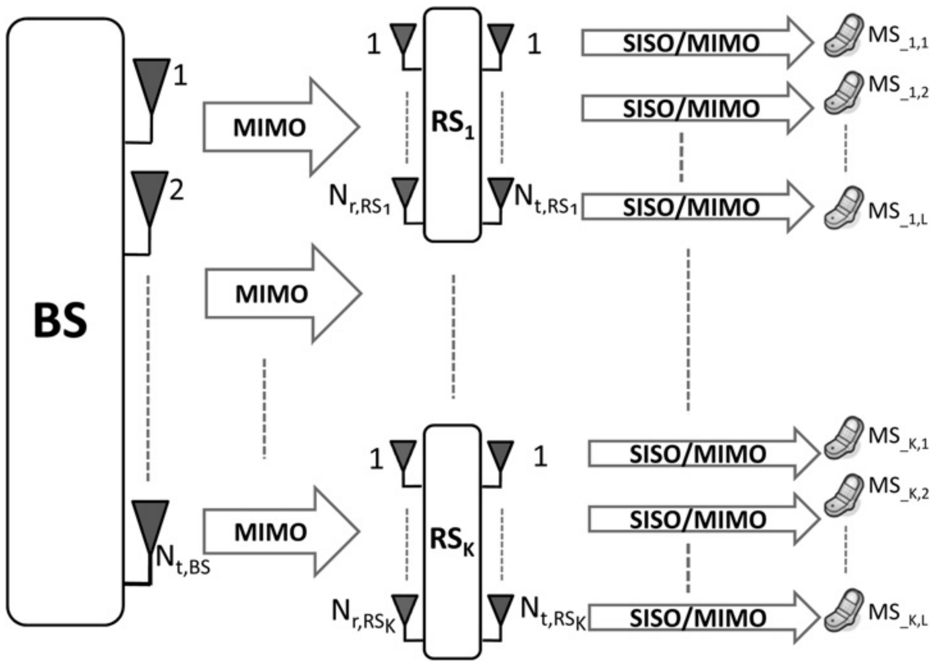


**Fig. 3** *Massive MU-MIMO system employing relay multiplexing*

the $K$ parallel RSs, where $\boldsymbol{z}_j = \begin{bmatrix} z_{j,1} & z_{j,2} & \cdots & z_{j,N_{r,RS_j}} \end{bmatrix}^T$ is the data stream targeted to $RS_j$. $\boldsymbol{z}_j$ is multiplied by $(N_{t,BS} \times N_{r,RS_j})$ a preprocessing matrix $W_{t_j}$ before its transmission to $RS_j$. The $N_{t,BS} \times 1$ symbol vector $\boldsymbol{x} = [x_1, \ldots, x_{N_{t,BS}}]$ transmitted by BS to $K$ RSs is given as

$$\boldsymbol{x}_j = \boldsymbol{W}_{t_j}\boldsymbol{z}_j, \quad j = 1, 2, \ldots, K \tag{15}$$

The $N_{t,BS} \times 1$ symbol vector $\boldsymbol{x} = [x_1, \ldots, x_{N_{t,BS}}]$, broadcast by the BS to $K$ RSs can be written as

$$\boldsymbol{x} = \sum_{j=1}^{K} \boldsymbol{x}_j = \sum_{j=1}^{K} \boldsymbol{W}_{t_j}\boldsymbol{z}_j \tag{16}$$

The symbol vector $\boldsymbol{y}_j$ received by $RS_j$ is given as

$$\begin{aligned} \boldsymbol{y}_j &= \boldsymbol{H}_j\boldsymbol{x} + \boldsymbol{n}_j = \boldsymbol{H}_j\boldsymbol{W}_{t_j}\boldsymbol{z}_j + \sum_{i=1, i\neq j}^{K} \boldsymbol{H}_j\boldsymbol{W}_{t_i}\boldsymbol{z}_i + \boldsymbol{n}_j \\ &= \boldsymbol{H}_j\boldsymbol{W}_{t_j}\boldsymbol{z}_j + \boldsymbol{H}_j\overline{\boldsymbol{W}}_{t_j}\bar{\boldsymbol{z}}_j + \boldsymbol{n}_j \end{aligned} \tag{17}$$

where $\boldsymbol{n}_j$ is $N_{r,RS_j} \times 1$-element vector representing AWGN with zero mean and unit variance. The parameter $\boldsymbol{H}_j\overline{\boldsymbol{W}}_{t_j}\bar{\boldsymbol{z}}_j$ represents the interference at $RS_j$. The parameters $\overline{\boldsymbol{W}}_{t_j}$ and $\bar{\boldsymbol{z}}_j$, respectively, represent the precoding matrices and data streams of all other RSs excluding $RS_j$ [15]. Multiple relay-access interference in (17) is eliminated by imposing the constraint $\boldsymbol{H}_j\boldsymbol{W}_{t_i} = 0$ for $i \neq j$ [16]. If $\overline{\boldsymbol{H}}_j$ is defined as

$$\overline{\boldsymbol{H}} = \begin{bmatrix} \boldsymbol{H}_1^T & \cdots & \boldsymbol{H}_{j-1}^T & \boldsymbol{H}_{j+1}^T & \cdots & \boldsymbol{H}_K^T \end{bmatrix}^T \tag{18}$$

then the elimination of interference in (17) requires $\boldsymbol{W}_{t_j}$ $(j = 1, \ldots, K)$ to be in the null space of $\overline{\boldsymbol{H}}_j$. If $\overline{\boldsymbol{V}}_{jn}$ denotes the null space of $\overline{\boldsymbol{H}}_j$, then the precoding matrix $\boldsymbol{W}_j$ for the data stream transmitted to $RS_j$ can be constructed by the columns of $\overline{\boldsymbol{V}}_{jn}$ to ensure zero interference at $RS_j$. Hence the signal received at the $RS_j$ can be expressed as

$$\boldsymbol{y}_j = \boldsymbol{H}_j\overline{\boldsymbol{V}}_{jn}\boldsymbol{z}_j + \boldsymbol{n}_j = \boldsymbol{F}_j\boldsymbol{z}_j + \boldsymbol{n}_j \tag{19}$$

where $\boldsymbol{F}_j = \boldsymbol{H}_j\overline{\boldsymbol{V}}_{jn}$ is the effective channel between BS and $RS_j$.

### 3.1 SEP analysis for first-hop detection at RS

The MLD at $RS_j$ is given as

$$\tilde{\boldsymbol{z}}_j = \underset{\boldsymbol{z}_j \in \mathcal{X}}{\operatorname{argmin}} \sum_{k=1}^{N_{r,RS_j}} \left| y_{j,k} - \sum_{l=1}^{N_{t,BS}/K} f_{j,k,l}z_{j,l} \right|^2 \tag{20}$$

The total number of symbols transmitted by the BS to $K$ RSs is $N_{t,BS}$ and $N_{t,BS}/K$ are the number of symbols transmitted towards $RS_j$. The effective channel gain at the $k$th receive antenna of $RS_j$ for the $l$th received symbol is $f_{j,k,l}$. It also represents the $(k, l)$ element of the effective channel matrix $F_j$. $\tilde{\boldsymbol{z}}_j$ denotes the decoded symbol vector for $\boldsymbol{z}_j = [z_{j,1} \cdots z_{j,(N_{t,BS}/K)}]^T$ and $\tilde{z}_{j,l}$ denotes the decoded symbol for $z_{j,\,l}$. Note that $\boldsymbol{z}_j$ is the sub-stream transmitted to $RS_j$ and $z_{j,l}$ is the $l$th symbol of $\boldsymbol{z}_j$. If $\bar{\boldsymbol{z}}_{j,l} = \{z_{j,1}, \ldots, z_{j,l-1}, z_{j,l+1}, \ldots, z_{j,(N_{t,BS}/K)}\}$ represents the set of transmitted symbols to $RS_j$ excluding the $l$th symbol $z_{j,l}$, then we can write the SEP of $z_{j,\,l}$ as a combination of SEPs conditioned on a set of error events corresponding to $\bar{\boldsymbol{z}}_{j,l}$ and their probabilities, similar to (3). Now following the same procedure as in Section 2, the average SEPs conditioned on no error $P(\tilde{z}_{j,l} \neq z_{j,l}|\text{err}(\phi))$, one error $P(\tilde{z}_{j,l} \neq z_{j,l}|\text{err}(k))$, two errors $P(\tilde{z}_{j,l} \neq z_{j,l}|\text{err}(l, m))$ and all errors $P(\tilde{z}_{j,l} \neq z_{j,l}|\text{err}(\Omega))$ in other symbols transmitted towards the $RS_j$ can be found. The SEP of each transmitted symbol is the same under i.i.d. fading case, when power is equally divided among all the transmitted symbols at the


*IET Commun.*, 2014, Vol. 8, Iss. 10, pp. 1788–1799
doi: 10.1049/iet-com.2013.1000

BS. Thus the total SEP at $RS_j$, $P(\mathrm{err})_j$, is given as

$$P(\mathrm{err})_j = \sum_{i=0}^{[N_{t,BS}/K]-1} \binom{[N_{t,BS}/K]-1}{i} \times \beta_j P(\mathrm{err})_j^i \left(1 - P(\mathrm{err})_j\right)^{[N_{t,BS}/K]-1} \quad (21)$$

The SEP $P(\mathrm{err})_j$ at $RS_j$ can be obtained by solving (21) for $P(\mathrm{err})_j$. Then the average SEP for the first-hop transmission, $P(\mathrm{err})_{\mathrm{hop}_1}$, can be written as

$$P(\mathrm{err})_{\mathrm{hop}_1} = \sum_{j=1}^{K} P(z_j) P(\mathrm{err})_j \quad (22)$$

where $P(z_j)$ denotes the probability that the sub-stream $z_j$ is transmitted and $P(\mathrm{err})_j$ represents the SEP of the substream $z_j$ at $RS_j$. When all sub-streams transmitted by the BS are of equal length, and they experience similar fading, the average SEP of the first-hop transmission can be expressed as

$$P(\mathrm{err})_{\mathrm{hop}_1} = \frac{1}{K} \sum_{j=1}^{K} P(\mathrm{err})_j \quad (23)$$

### 3.2 SEP analysis for second-hop detection at MS

In the second hop $K$ data streams are received at the RSs and are decoded. Each RS then forwards its data to the $L$ MSs it serves using MU-MIMO precoding techniques. As the total number of antennas of all the MSs served by $RS_j$ is equal to the number of antennas at $RS_j$, null-space-based precoding can be used at the $RS_j$ to mitigate multiple MSs access interference. The channel from $RS_j$ to $L$ MSs shown in Fig. 3 is partitioned into $L$ single-input single output (SISO) or MIMO parallel links depending on the number of antennas at the MSs. The decoded symbol vectors $\hat{z}_j$ at $RS_j$ are multiplied by a precoding matrix $R_{t_j}$ before its transmission to the MSs. $R_{t_j}$ can be designed by using null-space-based precoding, as described in Section 3. Then following a similar approach as in Sections 3 for first-hop transmission, the SEP for the second-hop transmission from $RS_j$ to the $l$th MS $P(\mathrm{err})_{RS_j,MS_l}$ can be expressed using (21). Then the overall SEP at the $l$th MS can be expressed using the following equation

$$P(\mathrm{err})_{MS_l} = P(\mathrm{err})_j + P(\mathrm{err})_{RS_j,MS_l} - P(\mathrm{err})_j P(\mathrm{err})_{RS_j,MS_l} \quad (24)$$

## 4 Relay-multiplexing system with SVD-assisted MU-MIMO precoding

The second approach, for the relay-access problem under consideration involves using SVD-based MU-MIMO precoding, introduced in [17], where the precoding matrices corresponding to the signal space of each RS are employed at the BS for transmitting data in parallel to multiple RSs. At the RS post-processing of received data is carried out to extract the data streams of different RSs. This strategy removes both MU-interference of the multiple RSs, and the inter-antenna interference (IAI) of the MIMO antennas of each RS. Although this approach is very interesting for relay-multiplexing scheme, we did not find an analytical derivation of its SEP performance in the literature for point-to-multipoint applications, even though there are some results for point-to-point case [18]. In this paper, we derive the SEP performance of SVD-based MU-MIMO precoding when employed for the proposed relay-multiplexing scheme (point-to-multipoint).

In this section we present the SEP analysis of the proposed relay-multiplexing method, employing SVD-assisted MU-MIMO precoding at the BS [17], and multiple antennas at the RSs. Consider that the relay-assisted cellular system shown in Fig. 1 assumes the configuration shown in Fig. 3, where the BS equipped with $N_{t,BS}$ antennas is communicating with $K \times L$ MSs, each equipped with $N_{r,MS_{j,l}}$ antennas, via $K$ multi-antenna RSs. It is assumed that $RS_j$ has $N_{r,RS_j}$ receiving and $N_{t,RS_j}$ transmitting antennas such that $N_{t,BS} = \sum_{j=1}^{K} N_{r,RS_j} = \sum_{j=1}^{K} N_{t,RS_j} = \sum_{j=1}^{K} \sum_{l=1}^{L} N_{r,MS_{j,l}}$ and that DF protocol is employed at all the $K$ RSs. The channel between BS and $K$ RSs is partitioned into parallel multiuser SISO channels or relay access using SVD-assisted MU-MIMO precoding at the BS and post-processing at $RS_j$. The parameter $\boldsymbol{H}_j$ denotes the channel matrix from the BS to $RS_j$, whereas $\boldsymbol{G}_{l,j}$ denotes the channel matrix from $RS_j$ to the $l$th MS. We assume that the BS and $RS_j$ have accurate knowledge of their forward channels. The BS broadcasts $K$ independent data streams $z_1, z_2, \ldots, z_K$ towards the $K$ parallel RSs, where $\boldsymbol{z}_j = \begin{bmatrix} z_{j,1} & z_{j,2} & \cdots & z_{j,N_{r,RS_j}} \end{bmatrix}^{\mathrm{T}}$ is the data stream transmitted to $RS_j$. $\boldsymbol{z}_j$ is multiplied by $(N_{t,BS} \times N_{r,RS_j})$ a preprocessing matrix $\boldsymbol{W}_{t_j}$ before its transmission to $RS_j$. The $N_{t,BS} \times 1$ symbol vector $\boldsymbol{x} = \left[x_1, \ldots, x_{N_{t,BS}}\right]$ transmitted by BS to $K$ RSs is given as

$$\boldsymbol{x} = \sum_{j=1}^{K} \boldsymbol{x}_j = \sum_{j=1}^{K} \boldsymbol{W}_{t_j} \boldsymbol{z}_j = \boldsymbol{W}_t \boldsymbol{z} \quad (25)$$

where $\boldsymbol{W}_t$ is an $(N_{t,BS} \times \sum_{j=1}^{K} N_{r,RS_j})$-element matrix given as

$$\boldsymbol{W}_t = [\boldsymbol{W}_1, \boldsymbol{W}_2, \ldots, \boldsymbol{W}_K] \quad (26)$$

and $\boldsymbol{z} = [z_1, z_2, \ldots, z_K]^{\mathrm{T}}$. The $N_{r,RS_j} \times 1$ vector $\boldsymbol{y}_j$ received at $RS_j$ is expressed as

$$\boldsymbol{y}_j = \boldsymbol{H}_j \boldsymbol{x} + \boldsymbol{n}_j = \boldsymbol{H}_j \boldsymbol{W}_t \boldsymbol{z} + \boldsymbol{n}_j = \boldsymbol{H}_j \boldsymbol{W}_{t_j} \boldsymbol{z}_j + \sum_{i=1, i \neq j}^{K} \boldsymbol{H}_j \boldsymbol{W}_{t_i} \boldsymbol{z}_i + \boldsymbol{n}_j \quad (27)$$

where $\boldsymbol{n}_j$ is $(N_{r,RS_j} \times 1)$-element AWGN vector with zero mean and unit variance. After computing the SVD of $\boldsymbol{H}_j$ and following the similar procedure as in [17], the received signal $\boldsymbol{y}_j$ at $RS_j$ can be expressed as

$$\boldsymbol{y}_j = \boldsymbol{U}_j \boldsymbol{\Lambda}_j^{1/2} \boldsymbol{V}_{js}^{\mathrm{H}} \boldsymbol{W}_t \boldsymbol{z} + \boldsymbol{n}_j, \quad j = 1, 2, \ldots, K \quad (28)$$

where $\boldsymbol{U}_j$ is an $(N_{r,RS_j} \times N_{r,RS_j})$-element unitary matrix, $\Lambda_j$ is an $(N_{r,RS_j} \times N_{r,RS_j})$-element diagonal matrix containing the eigenvalues of $\boldsymbol{H}_j^{\mathrm{H}} \boldsymbol{H}_j$ and $\boldsymbol{V}_{js}$ is an $(N_{t,BS} \times N_{t,BS})$-component matrix containing the eigenvectors corresponding to the non-zero eigenvalues of $\boldsymbol{H}_j^{\mathrm{H}} \boldsymbol{H}_j$. Now if we collect all the signal vectors $\{\boldsymbol{y}_j\}$ of all RSs into a vector $\boldsymbol{y} = [\boldsymbol{y}_1^{\mathrm{T}}, \ldots, \boldsymbol{y}_K^{\mathrm{T}}]^{\mathrm{T}}$, then according to (28) the overall received signal vector $\boldsymbol{y}$ of all $K$ RSs can be expressed as

$$\boldsymbol{y} = \boldsymbol{U}\boldsymbol{\Lambda}^{1/2}\boldsymbol{V}_s^{\mathrm{H}}\boldsymbol{W}_t\boldsymbol{z} + \boldsymbol{n} \tag{29}$$

where $\boldsymbol{U}$, $\boldsymbol{\Lambda}$ and $\boldsymbol{V}_s^{\mathrm{H}}$ are obtained from $\boldsymbol{U}_j$, $\boldsymbol{\Lambda}_j$ and $\boldsymbol{V}_{js}^{\mathrm{H}}$ according to [17, (25)]

$$\begin{aligned} \boldsymbol{U} &= \mathrm{diag}\{\boldsymbol{U}_1, \boldsymbol{U}_2, \ldots, \boldsymbol{U}_K\} \\ \boldsymbol{\Lambda} &= \mathrm{diag}\{\boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_K\} \\ \boldsymbol{V}_s &= \mathrm{diag}\{\boldsymbol{V}_{1s}, \boldsymbol{V}_{2s}, \ldots, \boldsymbol{V}_{Ks}\} \end{aligned}$$

The BS's preprocessing matrix $\boldsymbol{W}_t$ is designed such that the relay-access interference can efficiently be removed. According to (29), the relay-access interference can be efficiently removed when the transmit preprocessing matrix at the BS, $\boldsymbol{W}_t$, is chosen as

$$\boldsymbol{V}_s^{\mathrm{H}}\boldsymbol{W}_t = \beta \tag{30}$$

where the power allocation matrix $\beta = \mathrm{diag}\{\beta_1, \ldots, \beta_K\} = \mathrm{diag}\{\beta_{11}, \ldots, \beta_{1,N_{\mathrm{r,RS}_1}}; \ldots; \beta_{K1}, \ldots, \beta_{K,N_{\mathrm{r,RS}_1}}\}$ represents the transmission constraint. A fair power allocation scheme is to provide the same power to each data stream. So the coefficients $\beta_i$ are set as $\beta_1 = \beta_2 = \cdots = \beta_{\sum_{j=1}^{K} N_{\mathrm{r,RS}_j}} = \beta$ [1, 3], where $\beta$ is a constraint given as in [17, 19]. To satisfy (30), $\boldsymbol{W}_t$ can be set to

$$\boldsymbol{W}_t = [\boldsymbol{V}_s^{\mathrm{H}}]^{+}\beta = \tilde{\boldsymbol{W}}_t\beta \tag{31}$$

where $[\boldsymbol{V}_s^{\mathrm{H}}]^{+}$ represents the pseudoinverse of $\boldsymbol{V}_s^{\mathrm{H}}$. Now by substituting the preprocessing matrix of (31) into (29), the overall received vector $y$ of all the $K$ RSs can be written as

$$\boldsymbol{y} = \boldsymbol{U}\boldsymbol{\Lambda}^{1/2}\beta + \boldsymbol{n} \tag{32}$$

Specifically, the $N_{\mathrm{r,RS}_j}$-length received symbol vector of the $\mathrm{RS}_j$ can is given

$$\boldsymbol{y} = \boldsymbol{U}_j\boldsymbol{\Lambda}_j^{1/2}\beta_j\boldsymbol{z}_j + \boldsymbol{n}_j \tag{33}$$

Thus the $\mathrm{RS}_j$ experiences no relay-access interference offered by other RSs. However, there may also exist IAI among the antenna-specific symbols received at $\mathrm{RS}_j$. This IAI can effectively be removed by the SVD-based matrices $\boldsymbol{U}_j$. Hence, after post-processing the received signal vectors $\boldsymbol{y}_j$ by $\boldsymbol{U}_j^{\mathrm{H}}$, the relay-specific decoded variables can be expressed as

$$\tilde{\boldsymbol{y}}_j = \boldsymbol{\Lambda}_j^{1/2}\beta_j\boldsymbol{z}_j + \boldsymbol{U}_j^{\mathrm{H}}\boldsymbol{n}_j = \boldsymbol{\Lambda}_j^{1/2}\beta_j\boldsymbol{z}_j + \tilde{\boldsymbol{n}}_j \tag{34}$$

where $\tilde{\boldsymbol{n}}_j = \boldsymbol{U}_j^{\mathrm{H}}\boldsymbol{n}_j$ and $\boldsymbol{\Lambda}_j = \mathrm{diag}\{\lambda_{j,1}, \ldots, \lambda_{j,N_{\mathrm{r,RS}_j}}\}$ are the eigenvalues of $\boldsymbol{H}_j\boldsymbol{H}_j^{\mathrm{H}}$.

### *4.1 SEP analysis for first-hop detection at the $\mathrm{RS}_j$*

If we define $N_{\min} = \min(N_{\mathrm{t,BS}}, N_{\mathrm{r,RS}_j})$ and $N_{\max} = \max(N_{\mathrm{t,BS}}, N_{\mathrm{r,RS}_j})$, then the $N_{\mathrm{t,BS}} \times N_{\mathrm{r,RS}_j}$ MIMO channel between the BS and $\mathrm{RS}_j$ is decomposed into $N_{\min}$ SISO channels expressed as

$$\tilde{y}_{j,l} = \sqrt{\lambda_{j,l}}\beta_{j,l}z_{j,l} + \tilde{n}_{j,l}, \quad l = 1, 2, \ldots, N_{\mathrm{r,RS}_j} \tag{35}$$

The co-variance matrix of the transmit data vector $\boldsymbol{z}_j$ is an $N_{\min} \times N_{\min}$ matrix with off-diagonal entries equal to zero. The diagonal entries of this matrix represent the transmit power of each SISO channel, that is $P_{j,l}$, as

$$E\{\boldsymbol{z}_j\boldsymbol{z}_j^{\mathrm{H}}\} = \mathrm{diag}\{P_{j,1}, \ldots, P_{j,N_{\min}}\} \tag{36}$$

The total power constraint for the $\mathrm{RS}_j$ is $\sum_{l=1}^{N_{\min}} P_{j,l} = P_{j,\mathrm{T}}$, where $P_{j,\mathrm{T}}$ is the total power available at the BS for $\mathrm{RS}_j$. Furthermore, $\sum_{j=1}^{K} P_{j,\mathrm{T}} = P_{\mathrm{T}}$, where $P_{\mathrm{T}}$ is the total power available at the BS. If we let $\boldsymbol{s}_j = [s_{j,1}, \ldots, s_{j,N_{\min}}]^{\mathrm{T}}$ denote the equivalent transmit data vector towards $\mathrm{RS}_j$, with zero mean and unit variance, that is $E\{\boldsymbol{s}_j\boldsymbol{s}_j^{\mathrm{H}}\} = I_{\min}$, then

$$\boldsymbol{z}_{j,l} = \sqrt{P_{j,l}}\boldsymbol{s}_{j,l} \tag{37}$$

Thus using (37) in (35)

$$\tilde{y}_{j,l} = \sqrt{\lambda_{j,l}P_{j,l}}\beta_{j,l}s_{j,l} + \tilde{n}_{j,l} \tag{38}$$

The signal and the noise powers of the $l$th SISO channel of $\mathrm{RS}_j$ are $\lambda_{j,l}P_{j,l}\beta_{j,l}^2$ and $\sigma_{j,l}^2$, respectively. Thus, the SNR of the $l$th SISO channel of $\mathrm{RS}_j$ with equal power allocation, that is $P_{j,\,l} = P_{j,\mathrm{T}}/N_{\min}$, $\forall l$, can be expressed as

$$\begin{aligned} \gamma_{j,l} &= \frac{\lambda_{j,l}P_{j,l}\beta_{j,l}^2}{\sigma_{j,l}^2} = \frac{\lambda_{j,l}P_{j,\mathrm{T}}\beta_{j,l}^2/N_{\min}}{\sigma_{j,l}^2} \\ &= \frac{\lambda_{j,l}}{\left(N_{\min}/P_{j,\mathrm{T}}\beta_{j,l}^2\right)\sigma_{j,l}^2} \end{aligned} \tag{39}$$

For 4QAM modulated signal, SEP at $\mathrm{RS}_j$, $P(\mathrm{err})_j$, is given as

$$\begin{aligned} P(\mathrm{err})_j &= 2E_{\lambda_{j,l}}\left[Q\left(\sqrt{\gamma_{j,l}}\right) - \frac{1}{2}Q^2\left(\sqrt{\gamma_{j,l}}\right)\right] \\ &= 2E_{\lambda_{j,l}}\left[Q\left(\sqrt{\frac{\lambda_{j,l}}{N_{\min}\sigma_{j,l}^2/P_{j,\mathrm{T}}\beta_{j,l}^2}}\right)\right. \\ &\qquad \left. - \frac{1}{2}Q^2\left(\sqrt{\frac{\lambda_{j,l}}{N_{\min}\sigma_{j,l}^2/P_{j,\mathrm{T}}\beta_{j,l}^2}}\right)\right] \end{aligned} \tag{40}$$

where $E_{\lambda_{j,l}}[\cdot]$ denotes the expectation operator conditioned on $\lambda_{j,\,l}$, which is a randomly selected variable from $\boldsymbol{\Lambda}_j$. Now we can write (40) as (see (41))

where $\xi_{j,l} = \left(1/N_{\min}\sigma_n^2/\beta_{j,l}^2P_{j,\mathrm{T}}\right)$ and $f_{\lambda_{j,l}}(\lambda_{j,l})$ is the p.d.f. of $\lambda_{j,\,l}$ that is the p.d.f. of randomly selected eigenvalues of the complex Wishart matrix $\boldsymbol{H}_j\boldsymbol{H}_j^{\mathrm{H}}$ given as in [20, 21]. Using

$$P(\mathrm{err})_j = 2\left[\int_0^\infty Q\left(\sqrt{\xi_{j,l}\lambda_{j,l}}\right)f_{\lambda_{j,l}}(\lambda_{j,l})\,\mathrm{d}\lambda_{j,l} - \frac{1}{2}\int_0^\infty Q^2\left(\sqrt{\xi_{j,l}\lambda_{j,l}}\right)f_{\lambda_{j,l}}(\lambda_{j,l})\,\mathrm{d}\lambda_{j,l}\right] \tag{41}$$



doi: 10.1049/iet-com.2013.1000

Gaussian $Q$ function, its square and the p.d.f. of $\lambda_{j,\,l}$ in (41) and simplifying using results from [22], we can write the SEP at $\mathrm{RS}_j$ in the first-hop transmission as (see (42))

where $\Delta = N_{\max} - N_{\min}$ and $b = \Delta + n + 1$. Now the average SEP for the first-hop transmission, $P(\mathrm{err})_{\mathrm{hop}_1}$, in the relay-multiplexing system employing SVD-assisted MU-MIMO precoding and detection can be written using (22). Assuming all substreams are of equal length, and they experience similar fading, the average SEP of the first-hop transmission in relay-multiplexing system employing SVD-assisted MU-MIMO precoding and detection can also be written as in (23).

### 4.2 SEP analysis for second-hop detection at MS

In the second hop $K$ data streams are received at the RSs and are decoded. Each RS then forward its data to the $L$ MSs it serves using MU-MIMO precoding techniques. As the total number of antennas of all the MSs served by $\mathrm{RS}_j$ is equal to the number of antennas at $\mathrm{RS}_j$, SVD-assisted MU-MIMO precoding can again be used at the $\mathrm{RS}_j$ to mitigate multiple MSs access interference. The channel from $\mathrm{RS}_j$ to $L$ MSs is partitioned into $L$ parallel links. The decoded symbols vector $\hat{z}_j$ at $\mathrm{RS}_j$ is multiplied by a precoding matrix $R_{t_j}$ before its transmission to the MSs. $R_{t_j}$ can be designed by using SVD-assisted MU-MIMO precoding, as described in Section 4.1. Then following a similar approach as in Sections 4.1 for first-hop transmission, the SEP for the second-hop transmission from $\mathrm{RS}_j$ to the $l$th MS $P(\mathrm{err})_{\mathrm{RS}_j,\mathrm{MS}_l}$ can be expressed using (42). The overall SEP at the $l$th MS can be expressed using (24).

## 5 Capacity of relay-multiplexing scheme with massive MU-MIMO systems

This section presents the capacity analysis for MIMO relay-multiplexing scheme. As the first hop is divided into parallel MIMO links for the data transfer from the BS to $K$ RSs, the capacity of the $j$th MIMO link between the BS and $\mathrm{RS}_j$ can be expressed as

$$C_j = E\left[\log_2 \det\left(\boldsymbol{I} + \frac{\rho}{N_{\mathrm{t,BS}}}\boldsymbol{H}_j\boldsymbol{H}_j^{\mathrm{H}}\right)\right] \tag{43}$$

where $\rho$ denotes the SNR, and $\boldsymbol{H}_j$ is the channel between the BS and $\mathrm{RS}_j$. Then the capacity of the first-hop transmission is given by

$$C_{\mathrm{hop1}} = C_1 + C_2 + \cdots + C_K \tag{44}$$

As the second-hop transmission from $\mathrm{RS}_j$ to the $l$th MS is similar to the transmission from the BS to $\mathrm{RS}_j$, the capacity for the MIMO link from $\mathrm{RS}_j$ to the $l$th MS it serves is given by

$$C_{\mathrm{MS}_l} = E\left[\log_2 \det\left(\boldsymbol{I} + \frac{\rho}{N_{\mathrm{t,RS}_j}}\boldsymbol{G}_{l,j}\boldsymbol{G}_{l,j}^{\mathrm{H}}\right)\right] \tag{45}$$

where $\boldsymbol{G}_{l,\,j}$ is the channel between $\mathrm{RS}_j$ and the $l$th MS, $l = 1, 2, \ldots, L$. Then the capacity of the second-hop transmission between $\mathrm{RS}_j$ and all the MSs it serves is given by

$$C_{\mathrm{hop2,RS}_j} = C_{\mathrm{MS}_{j,1}} + C_{\mathrm{MS}_{j,2}} + \cdots + C_{\mathrm{MS}_{j,L}} \tag{46}$$

The over-all second-hop transmission capacity is thus given by

$$C_{\mathrm{hop2}} = C_{\mathrm{hop2,RS}_1} + C_{\mathrm{hop2,RS}_2} + \cdots + C_{\mathrm{hop2,RS}_K} \tag{47}$$

As the parallel RSs employ decode and forward protocol and the transmitted signal undergoes two states of decoding in cascade, the overall capacity of the two-hop relay-multiplexed massive MIMO system is given by

$$C = \min\left(C_{\mathrm{hop}_1}, C_{\mathrm{hop}_2}\right) \tag{48}$$

From (43) to (47), it is easily observed that the overall capacity of the system increases linearly with the number of RSs $K$, if other parameters are kept constant.

## 6 Simulation results

In this section we present simulation results to complement our analysis in the previous sections. In the simulations, 4QAM modulation is used throughout, and RSs are assumed to employ DF protocol. Flat Rayleigh fading is assumed for all the channel links, and the total power at the BS is set to unity and is divided equally among all the BS antennas. At the RSs, the sum of the transmitted power across all the parallel RSs is again set to unity and this power is also divided equally among all the RSs. Furthermore, the power available at each RS is divided equally among its transmitting antennas. Fig. 4 presents the SEP results for one-hop MIMO multiplexing system. The close agreement of our analytical analysis with the simulation results in this figure confirms the validity of our analysis. It is also obvious that the results for $2 \times 2$ MIMO system in this figure are in close agreement with the equivalent results in [1].

Next, we present simulation results for relay-multiplexing schemes with massive MIMO systems. In these simulations we consider two broad cases shown in Table 1. Case 1 assumes that all the RSs service one end user, corresponding to the case where relay multiplexing and massive MIMO antennas are employed for enhancing the data rate of an individual user. Case 2 assumes that the RSs service multiple independent users, corresponding to the case where relay multiplexing and massive MIMO antennas

$$P(\mathrm{err})_j = \frac{1}{\pi N_{\min}}\left[\sum_{r=0}^{N_{\min}-1}\sum_{m=0}^{r}\sum_{n=0}^{2m}(\Delta+n)!\left(\frac{(-1)^n(2m)!}{2^{2r-n}m!r!(\Delta+m)!}\binom{2r-2m}{r-m}\binom{2m}{2m-n}\right)\right.$$
$$\left.\times\left(\frac{3\pi}{4}+\sum_{p=1}^{b}(-1)^p\sum_{q=0}^{p-1}\frac{\xi_{j,l}^{p-q-1/2}}{(1+\xi_{j,l})^{p-1/2}}\binom{b}{p}\binom{p-1}{q}I\left(0,\,-\sqrt{\frac{1+\xi_{j,l}}{\xi_{j,l}}},\,1+q\right)\right)\right] \tag{42}$$


doi: 10.1049/iet-com.2013.1000

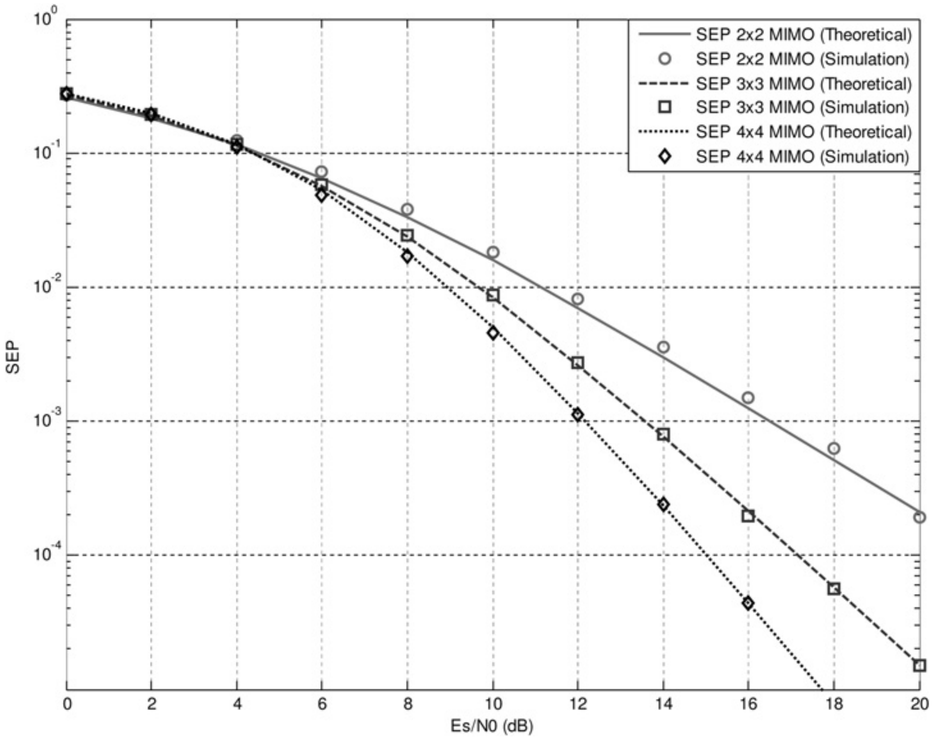


**Fig. 4** *SEP for 2 × 2, 3 × 3 and 4 × 4 MIMO multiplexing system*

**Table 1** Relay-multiplexing and massive MIMO schemes considered in the simulations

| | No. of parallel RSs | No. of transmit antennas at BS | No. of transmit antennas at *j*th RS | No. of receive antennas at MS | No. of MSs served |
|---|---|---|---|---|---|
| Case 1a | 2 | 4 | 2 | 4 | 1 |
| Case 1b | 2 | 6 | 3 | 6 | 1 |
| Case 1c | 2 | 8 | 4 | 8 | 1 |
| Case 1d | 2 | 10 | 5 | 10 | 1 |
| Case 2a | 2 | 20 | 10 | 1 | 20 |
| Case 2b | 2 | 24 | 12 | 1 | 24 |
| Case 2c | 2 | 30 | 15 | 1 | 30 |

are employed for increasing the numbers of users supported in the network. Our results for these two cases are summarised in the following.

In Fig. 5, we present summary of our SEP results for relay-multiplexing scheme employing null-space-based MU-MIMO precoding at the BS, for four different scenarios considered under case 1. The counterpart results for relay-multiplexing scheme employing SVD-assisted MU-MIMO precoding at the BS are shown in Fig. 6. It can be observed from these figures that our analytical results agree very closely with the simulations. It can also be observed, by comparing these two figures, that relay-multiplexing scheme employing SVD-assisted MIMO precoding slightly outperforms the relay-multiplexing scheme employing null-space-based MIMO precoding, for all cases simulated. However, both of these schemes have acceptable error rate performance and either of them can be a good choice for relay-multiplexing systems.

Fig. 7 presents the simulation results for case 1a in Table 1 for the scenario when the transmitted signals from the BS to the MS, via the RSs, are space–time block (STBC) encoded [23]. The transmitted signals from the BS are decoded at the RSs, re-encoded and re-transmitted to the MS. The results in this figure display the end-to-end SEP for the system, and it can be observed that the relay-multiplexing approach also works well for this scenario.

Fig. 8 presents the simulation results for three scenarios under case 2 in Table 1, for relay-multiplexing schemes employing massive MU-MIMO systems. In this simulation, null-space-based MU-MIMO precoding is employed for the first-hop transmission from BS to $K$ RSs whereas SVD-assisted MU-MIMO precoding is employed for the second-hop transmission from $K$ RS to $L$ MSs. It is observed from this figure that massive MU-MIMO systems assisted with relay multiplexing, can also be used in practice when the number of transmitting antennas are equal to the total number of users serviced. This is an encouraging new insight, since existing works on massive MIMO systems have mostly focused on the cases where BS antennas are much in excess of the number of end users serviced [24, 25]. Our results thus help to demonstrate that both of these precoding schemes are useful in relay-multiplexing schemes employing massive MIMO systems, where the number of transmitting antennas are equal to (or are not much in excess of) the total number of receiving antennas.

Fig. 9 presents the summary of the capacity enhancements offered by relay-multiplexing MU-MIMO systems considered



doi: 10.1049/iet-com.2013.1000

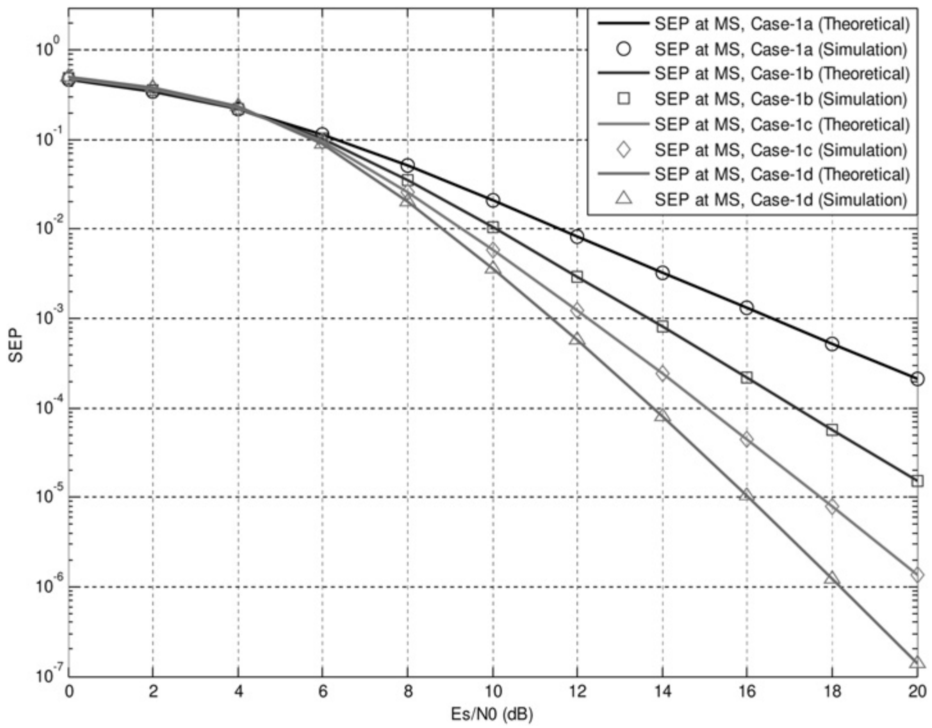


**Fig. 5** *Overall SEP for relay-multiplexing system employing null-space-based MIMO precoding when one end user is serviced and all MIMO antennas are used in multiplexing mode*

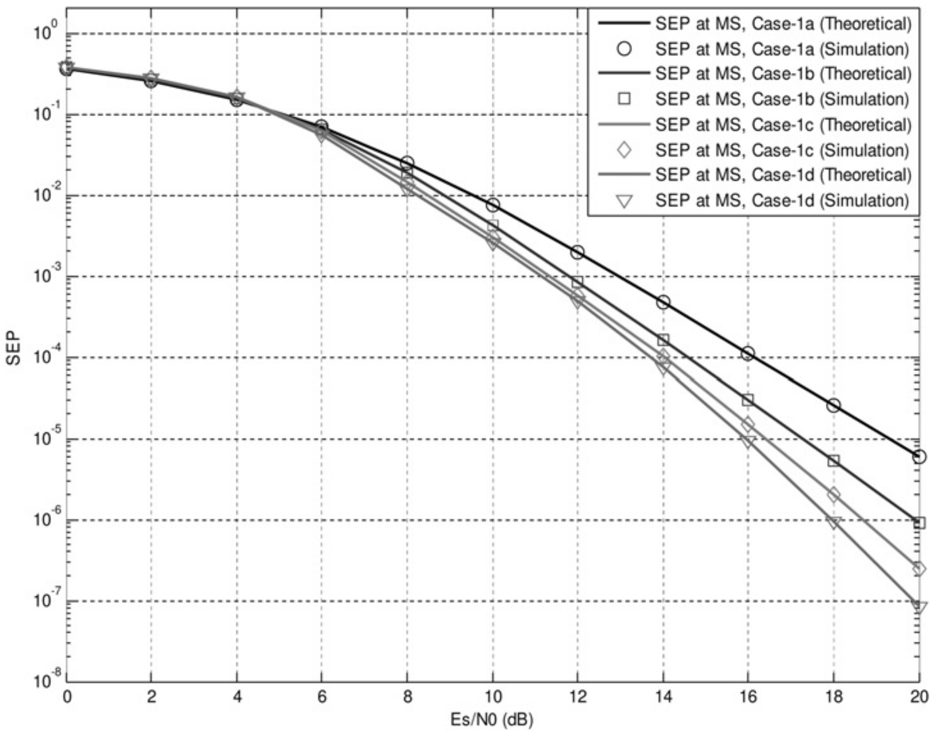


**Fig. 6** *Overall SEP for relay-multiplexing system employing SVD-assisted MIMO precoding when one end user is serviced and all MIMO antennas are used in multiplexing mode*

in Table 1, in comparison with the regular one- and two-hop MIMO systems. It is observed from this figure that relay-multiplexing approach provides capacity boost that grows linearly with the number of RSs, and when employed with massive MIMO techniques, tremendous boost in wireless capacity can be achieved.

## 7 Conclusions

This paper provides the SEP analysis of relay-multiplexing schemes employing massive (multiuser) MIMO systems. Using null-space-based and SVD-assisted MIMO precoding approaches, we derive closed-form expressions for the SEP


doi: 10.1049/iet-com.2013.1000

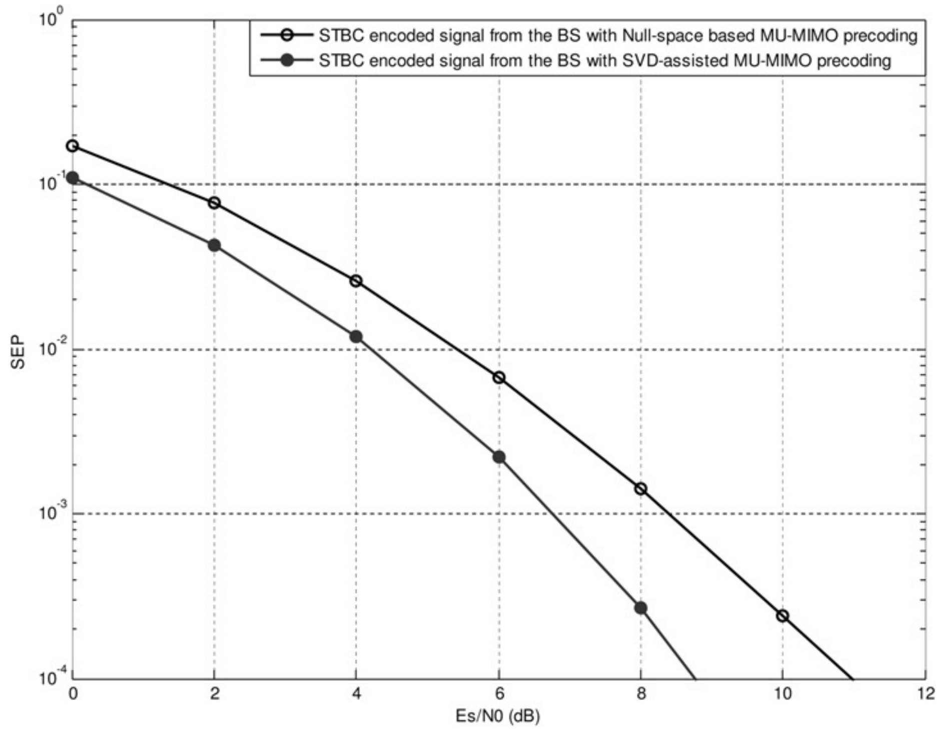


**Fig. 7** *Overall SEP for relay-multiplexing system when transmitting signals encoded with STBC from the BS and one end user is serviced*

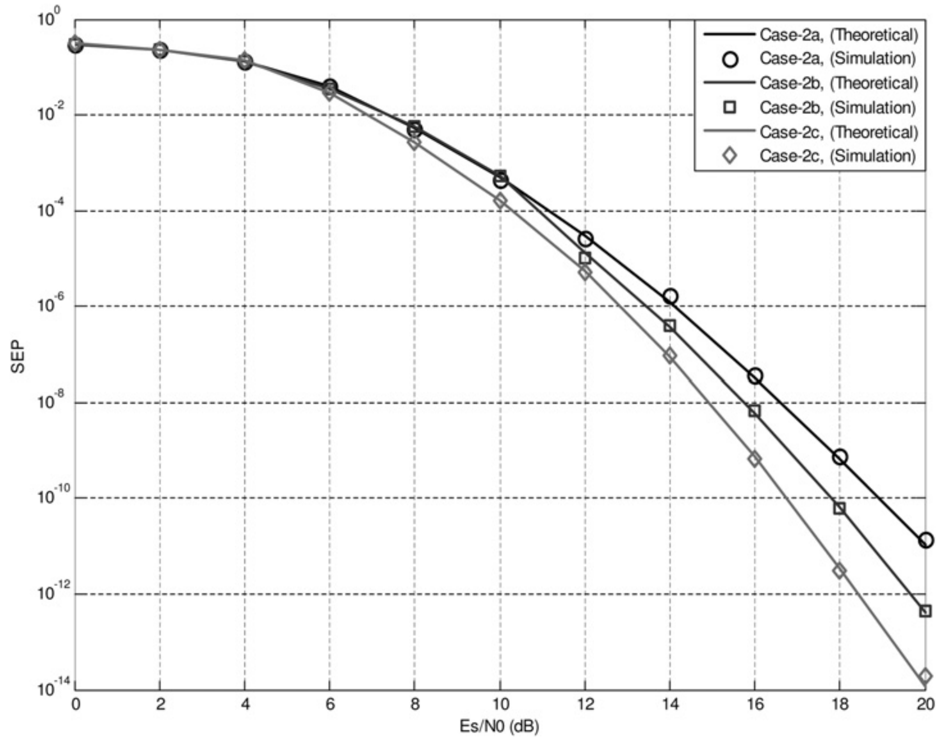


**Fig. 8** *Average SEP per user for relay-multiplexing scheme with massive MU-MIMO system when multiple users serviced are equal to the number of BS antennas*

performance of relay-multiplexing systems. The analytical results are verified by computer simulations. Based on our results, it is concluded that relay-multiplexing system when employed with proper MU-MIMO precoding scheme for multiuser-access interference rejection, can provide reliable, parallel and independent MIMO transmission links via the different RSs. It is also concluded that the relay-multiplexing approach employing SVD-assisted MIMO precoding scheme performs better than that of null-space-based MIMO precoding. However, both schemes have acceptable SEP performance in the SNR of interest in wireless communications. Thus, either of these schemes can be a good candidate for relay-multiplexing applications.



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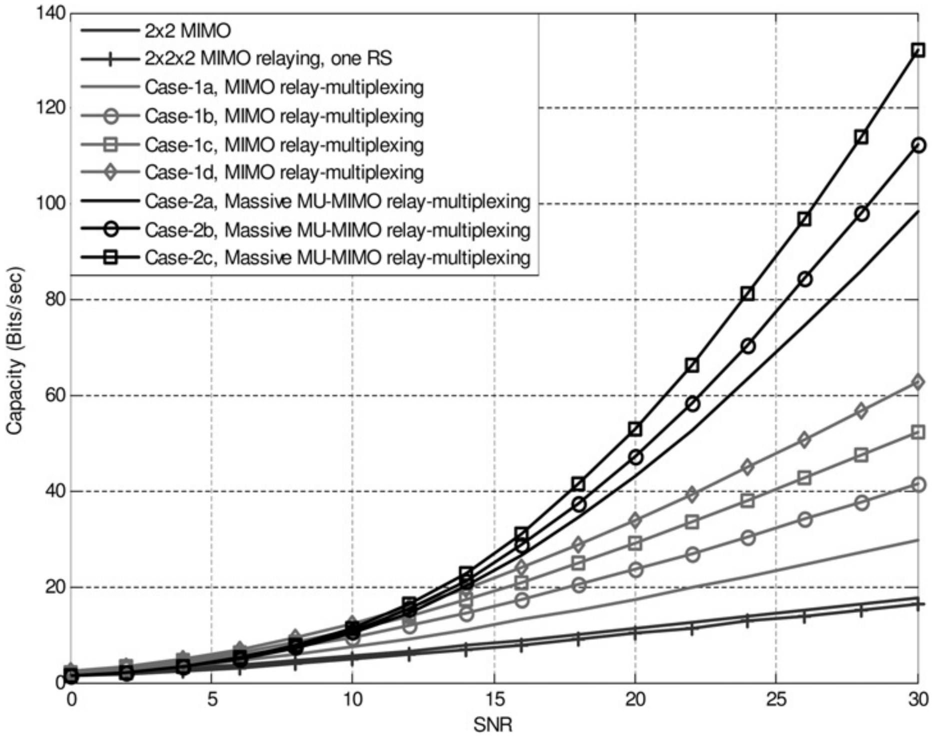


**Fig. 9** *Capacity enhancements using relay-multiplexing scheme with massive MIMO systems*


## 8 Acknowledgment

This work was sponsored by a grant from the National Plan for Science and Technology (NPST), King Saud University, grant no. 11-ELE1854-02.


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