Quantifying Connectivity of Grid-based Wireless Sensor Networks under Practical Errors

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***Abstract*—Grid-based deployments of Wireless Sensor Networks (WSNs) are widely used in a multiplicity of applications. However, practical factors such as communication irregularity and placement uncertainty have to be considered for more efficient deployments. In this paper, we examine connectivity properties of the 3D grid-based deployment when sensor placements are subject to random errors around their corresponding grid locations and hindrances to wireless communication channels exist. A generic approach is proposed to evaluate the average connectivity of the deployed network. This generic approach is independent of the grid-shape, random error distributions, and the environment wireless channel characteristics. The average connectivity is computed numerically and verified via extensive simulations. Based on the numerical results, quantified effects of positioning errors and grid edge length on the average connectivity are demonstrated. Furthermore, we discuss several ways of achieving efficient grid-based deployment planning for connectivity, and illustrate these approaches through numerical examples.**

Keywords-connectivity; random errors; grid deployment; WSNs.

I. INTRODUCTION

Wireless Sensor Networks (WSNs) are proving to be a promising technology in various environment monitoring domains. Sensor nodes in such wireless networks are used to sense physical and/or chemical properties in the surrounding environment. The readings are transmitted back to a sink node (base station) via wireless communication channels, called links, either periodically (sampling) or on-demand (event-based) to fulfill a specific task. However, this task may be very critical and require real-time interaction in order to deal with disasters, as in fire and pollution detection applications. Therefore, *connectivity* between the deployed sensor nodes and the sink node is of the utmost importance to ensure the timeliness of the measured data. The higher connectivity degree improves the communication capacity amongst the nodes. One of the most important factors that affects network connectivity is the deployment planning.

Practical deployment of the sensor nodes in environment monitoring applications is a challenging problem due to three main problems: 1) placement uncertainty, 2) communication range irregularity, and 3) required 3D setups. Placement uncertainty could occur because of timing errors in the placement, inaccurate distance estimation, and unexpected change in location due to weather conditions (rain and wind) and/or animals' visits. Meanwhile, communication irregularity in environment monitoring applications stems from natural and/or man-made obstacles in the terrain such as trees, hills, etc., in addition to extreme weather conditions, which may affect the propagated signals. Assuming a regular shape of the communication range is hence not accurate in practice. Flat and obstacle-free terrains are also highly unlikely in environment monitoring. Consequently, significant efforts are focusing nowadays on modeling irregular radio propagation and noisy communication channels in the past few years. Furthermore, sensor node deployment in environment monitoring applications becomes more challenging when nodes are not only placed at different variations in the horizontal plane, but also at different vertical levels (e.g. on trees, at soil surface and even underground). Existing deployment schemes for such applications [7] have not efficiently addressed the problem of connectivity in 3D space, which is a natural model in environmental applications.

One of the most effective and widely used deployment strategies in environment monitoring is the grid deployment [3]. Grid-based deployment can effectively limit the 3D search space of the candidate positions by placing the sensor nodes on well-organized vertices, in regular lattice structures. These vertices can be organized in different structures (e.g. cubes, octahedrons, pyramids, etc.) in 3D space to provide more accurate estimates in terms of the spatial properties of the data. We remark that while grid-based deployment in 3D space has been widely used in various environmental applications, the average connectivity of the grid when probabilistic communication model and placement uncertainty are assumed has not been investigated yet. In this paper, we examine connectivity properties of the 3D grid-based deployment under communication irregularity and placement uncertainty. We assume a probabilistic communication range, which has an arbitrary irregular shape in 3D space (due to presence of obstacles, and signal attenuation and reflection).

The major contributions of this paper are as follows. We provide a solid analytical derivation for the node positioning errors and the communication range irregularity on the grid. We propose a generic approach to evaluate the average connectivity in 3D grid-based deployments. This approach is applicable to various grid shapes and different kinds of random error distributions. Based on the proposed approach, solutions to other design problems, such as

978-1-4244-8389-1/10/$26.00 ©2010 IEEE

maximum error variance and maximum grid-edge length to satisfy a connectivity requirement, are hence devised.

The remainder of this paper is organized as follows. Section II outlines related work. Practical communication and error models are described in section III. A generic approach to measure node connectivity in grid-based deployments is proposed in section IV. In section V, we verify the correctness of the proposed approach, in addition to analyzing some practical grid-deployment problems and solutions. Finally, we conclude this paper in section VI.

II. RELATED WORK

In practice, deployment errors can affect coverage and/or connectivity of the grid. Coverage of the grid has been extensively studied in the literature and several approaches have been explored to overcome its problems. For example, properties of the grid under practical errors have been investigated in order to solve coverage problems before their occurrence. In reference [8], triangular grid-based deployment for coverage, when sensor placements are perturbed by bounded random errors around their corresponding grid vertices, is studied. Random errors are modeled by uniform displacements inside error disks of a given finite radius in 2D plane. The average coverage of the sensing field is derived as a function of the length of the grid edges and the radius of the random error disks.

On the other hand, extensive work has also been applied to overcome connectivity problems and failures during the deployment planning. However, the majority of this work is proposed to repair connectivity problems after their occurrence either by using node redundancy and/or node mobility, in addition to data fusion techniques [9]. Node redundancy in [4] is used to overcome disconnected networks. Redundant nodes are deployed and the ones that are not being used for communication or sensing are turned off. When the network becomes disconnected, one or more of the redundant nodes is turned on to repair connectivity. In reference [6], the lowest number of redundant nodes are added to a disconnected static network, so that the network remains connected. In [2], a distributed recovery algorithm is developed to address specific connectivity degree requirements. The idea is to identify the least set of nodes that should be repositioned in order to reestablish a particular level of connectivity. A shortcoming of such techniques is the requirement for extra nodes which may not be cost effective in environment monitoring applications. Also, when some redundant nodes fail, it may no longer be possible to repair the network. Meanwhile, data fusion techniques have been used to solve the problem of disconnected nodes without the need for redundant nodes by merging and aggregating the different readings of the sensors. In these techniques, data is stored locally when connectivity is disrupted, and is sent when connectivity is subsequently repaired [9]. A prominent problem with this approach is the latency in data transfer for time critical applications, such as fire and contaminates detection. Furthermore, node mobility can also be used to maintain network connectivity. Typically, mobile nodes are relocated after deployment to carry data between disconnected partitions of the network [5]. Providing radio connectivity using mobile nodes while considering ongoing missions, travelling distance and message exchange complexity has also been considered recently in [1]. However, relocating nodes without considering grid-connectivity properties and characteristics can have severe effects on the direction of movement and the choice of the most appropriate node to be moved.

Although the above techniques can aid with repairing connectivity problems, they do not address the source of these problems. Our work presents a radically different approach to address connectivity problems, complementing the work of the aforementioned techniques. In this paper, we address the properties of grid connectivity in practice and under realistic scenarios such as inaccurate positioning and communications irregularity. Thus, more efficient connectivity planning and maintenance can be achieved in terms of unnecessary redundant nodes, expensive mobility-dependent techniques and unwanted data delay in data fusion approaches.

III. SYSTEM MODELS

To quantify the grid connectivity, specific models which take into consideration the surrounding environment characteristics and the effects of placement errors have to be used. Such models are presented below.

A. *Communication Model*

In the literature, two types of communication models are widely used. The first assumes a binary disk to represent the communication range of a wireless device and send/receive signals only within the disk radius r_d. The second type of communication model assumes that the probability of communication between two wireless devices decays exponentially with distance.

In practice, wireless signals not only decay with distance, but also are attenuated and reflected by surrounding obstacles such as buildings, trees, etc. Hence, signal strength varies from one position to another based on the distance, and from one direction to another based on obstacles and terrain. Accordingly, the communication range of each device can be represented by an arbitrary shape in the 3D space. For realistic estimation of the arbitrary shape dimensions, we need a practical signal propagation model. This model can describe the path loss in the monitored environment by taking into consideration the effects of the surrounding terrain on the power (P_r) of received signals as follows [7].

$$P_r = K_0 - 10\gamma \log(d) - \mu d \qquad (1)$$

where d is the Euclidian distance between the transmitter and receiver, γ is the path loss exponent calculated based on experimental data, μ is a random variable describing signal attenuation effects[1] in the monitored site, and K_0 is a constant calculated based on the transmitter, receiver and field mean heights. Let P_r equal the minimal acceptable signal level to maintain connectivity. Assume γ and K_0 in Eq. (1) are also known for the specific site to be monitored. Thus, a probabilistic communication model which gives the probability that two devices separated by distance d can communicate with each other is given by

$$P_c(d,\mu) = Ke^{-\mu d^{\gamma}} \qquad (2)$$

where $K_0 = 10\log(K)$.

[1] Wireless signals are attenuated because of shadowing and multipath effects. This refers to the fluctuation of the average received power.

B. Placement Uncertainty Model

In this paper, we assume independent trivariate Normal errors in 3D space, around the targeted grid vertex. We assume that the error distribution has a mean of zero and is spherically symmetric in 3-axis Cartesian coordinates, centered at the grid vertex. Hence the probability density function is

$$f(x,y,z) = \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-(x^2+y^2+z^2)/2\sigma^2} \quad (3)$$

where σ^2 is the variance. To calculate the probability of connectivity between a sensor node placed at the point (u,v,q) and another sensor node placed inaccurately at (x,y,z) beside the grid vertex (x_i,y_i,z_i), assume e denotes the event that these two sensor nodes are connected. Then, the probability that the two sensors are connected, when they are subject to random Normal errors, is given by

$$P(e) = \int\int\int\int P_c(d_i,\mu) f(x,y,z) d\mu dx dy dz \quad (4)$$

where $P_c(d_i,\mu)$ is the probability that sensors at positions (u,v,q) and (x,y,z) will properly communicate with each other, when separated by a distance d_i with μ being a random variable representing the effects of multipath and shadowing. By plugging Eqs. (2) and (3) into Eq. (4), and assuming the path loss γ (=2), we can obtain the following form for $P(e)$

$$P(e) = \int \frac{K}{1+2\mu\sigma^2} e^{\frac{-\mu((u-x_i)^2+(v-y_i)^2+(q-z_i)^2)}{1+2\mu\sigma^2}} d\mu \quad (5)$$

Generally, the integrand in Eq. (5) is equal to zero if the distance between the two points (x_i,y_i,z_i) and (u,v,q) exceeds the communication range and/or the communication channel conditions prohibits the reception of wireless signals. Thus, $P(e)$ decreases dramatically as the distance between the grid vertices increases and the surrounding environment becomes more crowded by obstacles.

IV. QUANTIFYING GRID CONNECTIVITY

In this section, we propose a generic approach to derive the average connectivity percentage of the deployed sensor nodes on the 3D grid with the presence of random errors in placement. In addition, we consider a probabilistic communication model to determine connectivity between adjacent nodes on the grid vertices. We first define the following.

Definition 1 (Neighboring Set): The set of sensor nodes placed on grid vertices and connected to a common vertex at the center via a single grid edge from each sensor is the neighboring set of that central vertex.

Definition 2 (Neighboring Set Connectivity): The percentage of sensor nodes in a single neighboring set, which can communicate with the sink node via single or multiple hops, is called a neighboring set connectivity.

Note that the number of vertices (sensor nodes) in a neighboring set depends on the grid shape. For example, in a cubic grid a single neighboring set will have six vertices as shown in Figure 1. In the following, we analytically derive the connectivity of a single neighboring set (V). Suppose we have N sensor node distributed on the 3D grid vertices and each sensor node has a probabilistic communication range of an arbitrary shape. Let C_j denote the percentage of V, centered at vertex j, being connected to at least one node x_i. Assume x_i is a sink node, or a sensor node connected to the sink node via single or multiple hops. Then, the neighboring set connectivity is the expectation of C_j and is calculated by

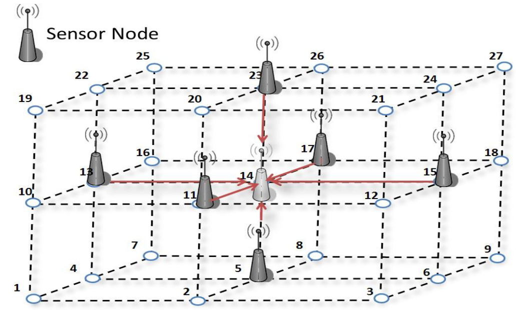

Figure 1. An example of a single neighboring set in cubic grid. The neighboring set of the centered node at vertex 14 consists of 6 sensor nodes distributed at vertices 5, 11, 13, 15, 17, and 23.

$$E[C_j] = \sum_V \frac{1}{v} p(u) \quad (6)$$

where $p(u)$ is the probability that the node at vertex $u \in V$ is at least connected to x_i and v is the number of neighboring grid vertices in the set V (e.g. v = 6 in cubic-grid). Let e_i, (i=1,…,N) denote the event that the node at vertex u is connected to x_i under random possible errors in placement. Then, we have

$$p(u) = P(e_1 \cup e_2 \cup \ldots \cup e_N) \quad (7)$$

Thus, we get

$$p(u) = \sum_i P(e_i) - \sum_{i<j} P(e_i)P(e_j) + .. + (-1)^{N+1} P(e_1)P(e_2)..P(e_N) \quad (8)$$

where $P(e_i)$ is calculated from Eq. (5) for random Normal errors. We remark that in typical grid deployments, even with random errors, a given sensor node at vertex u will be connected with a non-negligible probability only by those sensors that are the closest to u (i.e. neighboring set of u). Contrarily, the probability of being connected to sensors not in the neighboring set of u may be taken to be negligible. Therefore, many terms in Eq. (8) can be safely set to zero. By computing the summation of Eq. (8), for all $u \in V$, over v as noted by Eq. (6), we achieve the single neighboring set connectivity. To evaluate the grid connectivity, we compute the average connectivity percentage of the deployed network.

Definition 3 (Average Connectivity Percentage): The average connectivity percentage of the deployed wireless sensor network is the arithmetic average of all neighboring sets on the grid.

This arithmetic average is calculated by

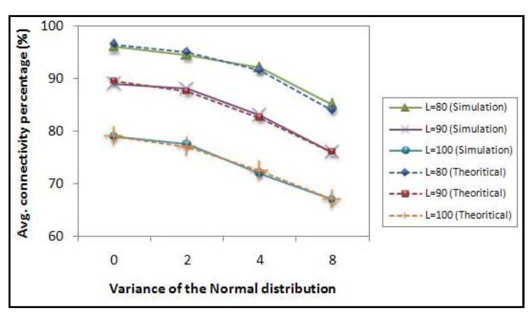

Figure 2. The average connectivity percentage vs. the variance of the Normal distribution.

$$Arithmatic\ Average = \frac{\sum_{j=1}^{N_s} E[C_j]}{N_s} \tag{9}$$

where N_s is the total number of neighboring sets of the deployed grid.

V. Discussion & Numerical Results

In this section, we quantify the resilience of grid-based deployments under practical settings. The average connectivity, in the presence of random errors derived above, is computed and compared to simulation results. 1000 deployment instances are randomly generated on cubic grid vertices, and hence $v = 6$. We distribute grid vertices in a 700*700*200 (m^3) monitored space. Each simulated deployment can have up to 110 sensor nodes (i.e. $N = 110$) with only one base station. Random Normal errors are applied on these sensor nodes while they are placed on grid vertices. Based on experimental measurements [7], we set the communication model variables to be as follows: $\gamma = 2$, $P_r = -104$ (dB), $K_0 = 42.152$, and μ to be a random variable that follows a log-normal distribution function with mean 0 and variance of 10. The simulator determines whether a sensor node is connected to its neighbors or not based on that probabilistic communication model.

The average connectivity of the grid is determined by the variance σ^2 of the Normal distribution and the length of the grid edge L. We first study how the variance of the Normal distribution impacts the average connectivity. Figure 2 depicts the impact of σ^2 on the average connectivity percentage. The results from theoretical derivations in section IV and from simulations match very well, and hence the correctness of our general approach is validated. Results of the rest of this section are numerically derived, based on the analytical study in section IV. Figure 3 plots the maximum σ^2 as a function of L. We notice that for 90% connectivity the maximum variance should not be more than 2 and the grid edge length should not exceed 80 (m). We also vary the value of L from 80 to 100 (m). The average connectivity of the grid as a function of L is presented in Figure 4. Unsurprisingly, the average connectivity of the grid is decreasing while the grid edge length is increasing. As the variance σ^2 is increasing, the grid average connectivity is decreasing. The average connectivity of the sensing field is a monotonically decreasing function of L. This characteristic can be used to describe the maximum L for a given σ^2 and a connectivity requirement. We analyze and describe the conditions when the required connectivity percentage is 80%, 85%, 90% and 95%. The results are plotted in Figure 5.

VI. Conclusion

In this paper, we investigated the connectivity properties of grid-based WSN deployment in the presence of inevitable random errors. These errors can cause nodes to be displaced from grid vertices and/or affect the communication channel. We proposed a generic approach for quantifying the average connectivity of the grid under practical random errors. This approach is applicable to a multiplicity of random error distributions and different grid shapes. We applied the generic approach to normally distributed errors in placement with the assumption of an arbitrary 3D shape for the communication range. Numerical results of the proposed approach demonstrate how resilient grid-deployment is to random placement errors and unpredictable communication ranges. Future work could investigate grid-connectivity under other types of positioning errors such as a priori known bounded errors.

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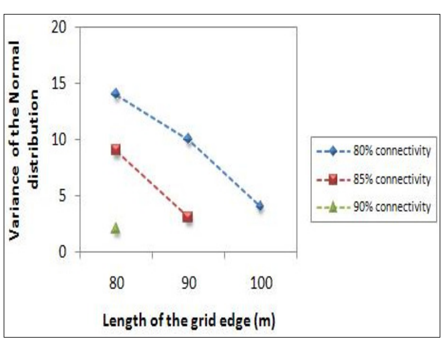

Figure 3. The maximum σ^2 vs. L.

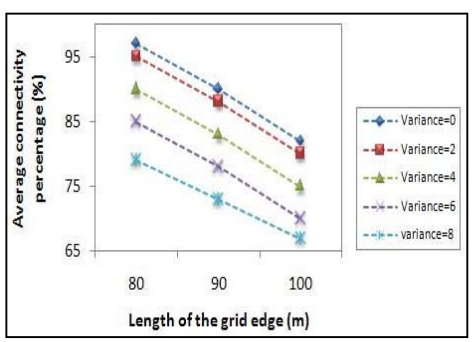

Figure 4. The average connectivity percentage vs. L.

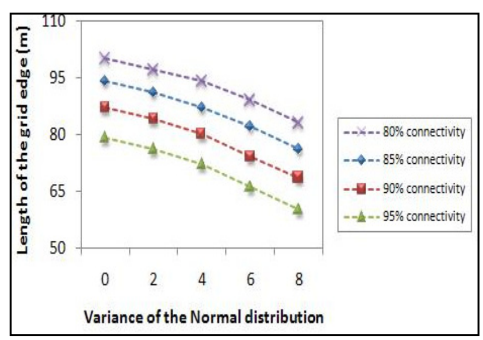

Figure 5. The maximum L vs. the σ^2.