This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2010 proceedings.

Symbol Loss Probability of OFDMA Technique in Mobile Multi-Hop Relaying Systems

Hassan A. Ahmed*, Ahmed Iyanda Sulyman•, and Hossam S. Hassanein+
*Electrical and Computer Engineering Department, •Electrical Engineering Department, +School of Computing
*+Queen's University, Canada, Kingston, K7M 3N6
•King Saud University, Saudi Arabia, Riyadh 11421
Emali:{*3haea@queensu.ca, •asulyman@ksu.edu.sa, +hossam@cs.queensu.ca}

***Abstract*— In this paper, we quantify the expected symbol loss probability in mobile multi-hop relaying systems employing Orthogonal Frequency Division Multiple Access (OFDMA) Techniques. We obtain the expected number of collisions and calculate the probability of symbol loss when a conflict occurs between subcarriers of two or more non-transparent relay stations, as well as we calculate the average symbol loss probability in the system. Also, the proportion of symbol with degraded SNR is measured and the collision rate is calculated. Our analysis shows that the resulting collision due to the simultaneous use of a subcarrier in different non-transparent relay stations can be very significant in a multi-hop relaying communication system, and severely degrades the SNR, and affects the QoS support in the system.**

***Index Terms*—Mobile Multi-hop relaying, WiMAX, OFDMA systems, Broadband wireless access.**

I. Introduction

Multi-hop relaying has been adopted in several wireless networks such as 3G cellular, WLANs and WiMAX systems as a cost-effective means of extending the coverage or increasing the capacity of the wireless system. For example, mobile multi-hop relay extension for IEEE 802.16e system is the subject of ongoing standardization activities within the IEEE 802.16j Task Group.

The emerging IEEE 802.16j standard [1], enhances the IEEE 802.16e PHY and MAC [2] to enable support of multi-hop communication between a mobile station (MS) and a base station (BS) through intermediate relay stations (RSs), which can be moible or fixed. Deploying RSs in the coverage area of BS as defined in IEEE 802.16j has been considered a promising solution that can replace the 802.16e mesh mode for coverage extension, throughput enhancement and overcoming coverage holes. In such a system the communication between MS and BS is done through two hops: first hop between MS and RS and second hop between RS and BS. Each RS amplify or decode users data and forward it to the BS. Two modes are proposed for relay operations in IEEE 802.16j standard, namely, transparent and non-transparent. In the former, the relay station operates in the same frequency band as the BS, while in the later the relay station operates in a different frequency band than the BS.

Orthogonal frequency division multiplexing (OFDM) is the defacto access mechanism for next generation broadband wireless access networks (BWANs). OFDM provides an efficient broadband data transmission by sending parallel data over a number of closely-spaced subcarriers. For multi-user support (OFDMA), it is desirable to divide the OFDM symbol into a number of subchannels, each consists of a number of subcarriers. One or more subchannels are assigned to one MS depends on the application (i., e., voice or data).

One of the limitations of WiMAX is frequency reuse because the channel can only be reused if the interference in the network is minimized. Frequency reuse increases interference by allowing more users to access the same channel. As the number of MSs increases in the system, more MSs can share a same OFDM symbol. This makes the OFDMA systems more sensitive to subcarrier collisions, hence increasing the level of interference.

Some proposals to reduce interference are available in the literature [3], [4], [5]. Cell sectorization and special channel allocation on the cell edges are some solutions for reducing the interference [3]. Fractional frequency reuse (FFR) is one scheme used to minimize interference [4]. Each cell sector is assigned a fraction (e., g., $1/3^{rd}$) of the available subcarriers. In this case, fewer subcarriers are used, therefore interference can be reduced. Frequency reuse becomes more complicated in a mobile multi-hop relay network, since more relay stations use the same frequencies, therefore, the interference level becomes high. An analytical model to capture and quantify the effect of collisions as a result of simulatinous use of subcarriers is still needed. For IEEE 802.16 networks, only interference calculations for single-hop based on simulations are available in the literature [5]. To the best of our knowledge there is no study to quantify collisions between subcarriers of OFDMA access systems in mobile multi-hop relay networks.

In this paper, we compute the expected number of collisions between two or more non- transparent relay stations, then we calculate the probability of symbol loss of OFDMA technique in mobile multi-hop relaying systems. It is worth to mention that not all symbols involved in a collision are lost, but collisions can increase the level of interference making the symbol loss probability high. The IEEE 802.16 standard [2], defines a threshold for the signal-to-noise-ratio (SNR) below which the bit-error-rate (BER) is unacceptable ($> 10^{-6}$). Therefore, the proportion of symbols with degraded SNR is measured and the collision rate is calculated in the case of two or more relay stations.

The remainder of this paper is organized as follows. Section II presents the system model. In Section III, the number of collisions resulting from the simultaneous use of subcarriers in two non-transparent relay stations is obtained. In Section IV, the expected number of collisions in the case of n non-transparent relay stations is obtained. Section V, presents the probability of symbol loss analysis in the system. Section VI, demonstrates some numerical results to validate the system performance analysis. Finally, Section VII concludes the paper.

II. SYSTEM MODEL

Consider a broadband wireless network employing OFDMA access mechanism over mobile multi-hop relaying channels, with each OFDM symbol consisting of N_c subcarriers. We assume that there are L MSs in the system uniformly distributed in the coverage area of the BS, and that each MS can be associated with the BS or a RS whichever provides stronger signal-to-noise ratio (SNR). An example of this model for the case $R = 2$ (two-hops relay network) is illustrated in Fig.1 for a cellular deployment, in which each cell is serviced by a BS, located at the center of the cell, and six RSs, each equidistant from the BS and located at the center of each side of the hexagon as shown.

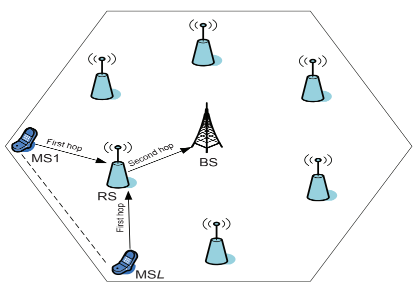

Fig. 1. Mobile Multi-hop Relay System

We model a wireless relaying system as presented in Fig.1, where the OFDM symbol consists of N_c subcarriers is divided into G groups (subcahnnels), each consists of N_c/G subcarriers. Frequency assignment to MSs is on the bases of subcahnnels by randomly picking a subcarrier from each group. This frequency allocation is made in each RS in a centralized manner (i., e., Base station), therefore no collisions are possible between MSs belong to the same RS. However, collisions are possible if nearby RSs use the same subcarriers. In this work, we calculate the expected number of collisions between subcarriers of n RSs, numbered from $0, \ldots, r-1$, where the target RS is numbered RS_0. Relay station RS_n contains S_n allocated subcahnnels and the vector $\mathbf{S} = (S_0, \ldots, S_{r-1})$ denotes the number of allocated subchannels in the r^{th} relay stations.

III. CASE OF TWO INTERFERING NON-TRANSPARENT RELAY STATIONS

In this section, the expected number of collisions resulting from the simultaneous use of subcarriers in two non-transparent relay stations is obtained.
The expected number of collisions in one group $g = 1$ for the case $n = 2$ relay stations, given the vector $\mathbf{S}$ of occupied subcarriers in this group is equal to

$$E_{\mathbf{S}}[g] = \widehat{E}_{\mathbf{S}}[g] \tag{1}$$

Then the total expected number of collisions in G groups can be expressed as

$$E_{\mathbf{S}}[g] = G.\widehat{E}_{\mathbf{S}}[g] \tag{2}$$

where $\widehat{E}_{\mathbf{S}}[g]$ is the expected number of collisions in one group can be given by

$$\widehat{E}_{\mathbf{S}}[g] = \sum_{i \epsilon I} i \widehat{P}_{\mathbf{S}}(i) \tag{3}$$

where $\widehat{P}_{\mathbf{S}}(i)$ is the probability of having i collisions in this group given by (4), The expected number of collisions in one group is then obtained by (2). The global number of collisions GI is calculated based on the collisions in each of the G group: $\mathbf{i} = (i_0, i_1, \ldots, i_{G-1})$, $GI = \sum_{s=0}^{G-1} i_s$. These collisions being independent and the groups identical, we obtain the expectations in (1).
In each group, there are S_n occupied subcarriers in relay n. We calculate the probability of having i collisions in this group. This is equivalent to the two relays choosing independently subcarriers S_0 and S_1 from the available N_c/G ones. There are $\binom{N_c/G}{S_0}$ possible combinations for relay RS_0 and the probability of choosing one of these combinations is equal to $1/\binom{N_c/G}{S_0}$. If the relay RS_0 had chosen a given set of S_0 subcarriers, the probability of i collisions can be expressed as

$$\widehat{P}_{\mathbf{S}}(i) = \frac{\binom{S_0}{i}\binom{N_c/G - S_0}{S_1 - i}}{\binom{N_c/G}{S_1}} \tag{4}$$

IV. CASE OF N INTERFERING NON-TRANSPARENT RELAY STATIONS

In this section, a general case $(n > 2)$ is considered, and the expected number of collisions resulting from the simultaneous use of subcarriers in n non-transparent relay stations is obtained.
Lemma 1: The probability of having i collisions in the case of n non-transparent relay stations in a given group can be expressed as

$$\widehat{P}_{\mathbf{S}}(i) = \binom{S_0}{i}\left[1 - \prod_{n=1}^{r-1} \frac{N_c/G - S_0 - S_n + i}{N_c/G - S_0 + i}\right]^i \prod_{n=1}^{r-1}\left[\prod_{m=0}^{S_n - 1} \frac{N_c/G - (S_0 - i) - m}{N_c/G - m}\right] \tag{5}$$

Proof of Lemma 1:
We first calculate the probability that $S_0 - i$ subcarriers in a given group belong to RS_0 do not experience collision. This can be possible only if we order the subcarriers, therefore the probability that the m^{th} subcarrier of RS_n does not collide

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2010 proceedings.

with the $S_0 - i$ subcarriers of RS_0, since the probability that the first m^{th} subcarriers do not collide can be expressed as

$$\left[\frac{N_c/G - (S_0 - i) - m}{N_c/G - m}\right] \tag{6}$$

Since the assigned subcarrier is randomly picked up from the remaining subcarriers $\frac{N_c}{G} - (S_0 - i) - m$, and there are only $\frac{N_c}{G} - m$ available subcarriers, the probability that $S_0 - i$ subcarriers do not experience collision can be expressed as

$$\widehat{q}_{\mathbf{S}}(S_0 - i) = \prod_{n=1}^{r-1} \prod_{m=0}^{S_n - 1} \left[\frac{N_c/G - (S_0 - i) - m}{N_c/G - m}\right] \tag{7}$$

The probability that i subcarriers of RS_0 collide, with the knowledge that the remaining $S_0 - i$ subcarriers do not collide based on the probability that at least one of these i subcarriers do not collide with the m^{th} RS_n subcarriers given by

$$\left[\frac{N_c/G - (S_0 - i) - m - 1}{N_c/G - (S_0 - i) - m}\right] \tag{8}$$

Assume that collisions are independent of each other, this probability can be given by

$$\begin{aligned}\widehat{P}_{\mathbf{S}}(i|S_0 - i) &= \left[1 - \prod_{n=1}^{r-1} \prod_{m=0}^{S_n - 1} \frac{N_c/G - (S_0 - i) - m - 1}{N_c/G - (S_0 - i) - m}\right]^i \\ &= \left[1 - \prod_{n=1}^{r-1} \frac{N_c/G - S_0 - S_n) + i}{N_c/G - S_0 + i}\right]^i \end{aligned} \tag{9}$$

Hence, considering the number of possible combinations, the probability of i collisions in the same group in the case of n relay stations can be calculated by (5). ∎

The probability obtained in (5) will be used to calculate the symbol loss probability and the SNR degradation in different non-transparent relay stations.

V. Analysis of Probability of Symbol Loss

Consider that we have L MSs uniformly distributed in the coverage area of the BS. These MSs are supporting C classes of calls. The arrival of class-s calls to relay n follow a Poisson process with arrival rate $\lambda_{s,n}$ and requires c_s subchannels. The service time has a mean $\frac{1}{\mu_{s,n}}$. When a class-s call arrives it will be accepted only if there are more than $N_c/G - c_s$ subcarriers available, otherwise it will be rejected. Let Vector $V_n = (V_{0,n}, \ldots, V_{C-1,n})$ represent the number of calls belonging to class C in relay n. Vector **S** represents the number of occupied subchannels in n relay given by

$$\mathbf{S} = \left[\sum_{s=0}^{C-1} c_s V_{s,n-1}\right] \tag{10}$$

The expected number of collisions is equal to

$$E[i] = \sum_S \left[\prod_{n=1}^{r-1} P_n(S_n)\right] E_{\mathbf{S}}[i] \tag{11}$$

where $E_{\mathbf{S}}[i]$ is the expected number of collisions given the number of occupied subchannels S_n obtained in (1). Where $P_n(S_n)$ is the probability of having collision with S_n occupied subchannels in relay n equal to

$$P_n(S_n) = \sum_{V_n} p_n(V_n) = \sum_{s=0}^{L-1} l_s V_{s,n} \tag{12}$$

where $p_n(V_n)$ is the probability of having $V_{s,n}$ class-s calls in relay n given by

$$p_n(V_n) = \frac{1}{Q} \prod_{s=0}^{I-1} \frac{(\frac{\lambda_{s,n}}{\mu_{s,n}})^{V_{s,n}}}{V_{s,n}!} \tag{13}$$

where Q is a normalizing constant given by

$$Q = \sum_{V_n} \prod_{s=0}^{I-1} \frac{(\frac{\lambda_{s,n}}{\mu_{s,n}})^{V_{s,n}}}{V_{s,n}!} \tag{14}$$

It is well known that at a given power and carrier frequency, the available data rate is inversely proportional to the distance from BS as a result of SNR degradation. Consider RS_n interferes with RS_0, the SNR at a given subcarrier can be obtained from the received signal on that subcarrier as follow [8].

$$Y_1[k] = H_0[k]H_1[k]X[k] + H_1[k]W_0[k] + W_1[k] \tag{15}$$

where $X[k]$ represents the transmitted symbol on subcarrier k, $H_0[k]$ and $H_1[k]$ represent the flat Rayleigh fading channel for the first and second hop, respectively. $\sigma^2_{W_0}$, and $\sigma^2_{W_1}$ are the variances of the additive white Gaussian noise in the first and second hop, respectively. Recall that there is no inter-carrier-interference (ICI) between subcarriers in the same group (subchannel). Therefore the SNR for a two hop network can be expressed as

$$SNR = \frac{E_s E[z^2]}{\sigma^2_{W_0[k]} + \sigma^2_{W_1[k]}} \tag{16}$$

where E_s denotes the transmitted power per subcarrier k, and $E[z^2] = E[H_0^2]E[H_1^2]$. The path loss p_n between relay station n and a given receiver [9], can be expressed as

$$p_n = d_n^{\mu} 10^{\frac{\xi_n}{10}} \tag{17}$$

where d_n denotes the distance from relay station n to the receiver, ξ_n is the log-normal random variable due to shadowing,

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2010 proceedings.

and $\mu = 4$. When a collision occurs at a given subcarrier, the bit-error-rate (BER) performance degrades below a specific target if the SNR goes below a given threshold ρ.
Using (15) and (17), the probability for a symbol to be lost when a collision occurs between RS_0 and RS_n can be expressed as

$$p_{0n} = P\Big(E_s(\frac{d_0}{d_n})^{\mu}10^{\frac{\xi_0-\xi_n}{10}} + W_0 d_0^{\mu}10^{\frac{\xi_0}{10}} + W_1 d_0^{\mu}10^{\frac{\xi_0}{10}} > \frac{E_s}{\rho}\Big) \quad (18)$$

At any MS's position, the shadowing can be modelled by a component ϵ common to all relay stations and another one ϵ_n for RS_n, and $\xi_n = x\epsilon + y\epsilon_n$, where $x^2 + y^2 = \frac{1}{2}$, (18) can be simplified as

$$p_{0n} = P\Big(E_s(\frac{d_0}{d_n})^{\mu}10^{\frac{y\epsilon_0-\epsilon_n}{10}} + W_0 d_0^{\mu}10^{\frac{\xi_0}{10}} + W_1 d_0^{\mu}10^{\frac{\xi_0}{10}} > \frac{E_s}{\rho}\Big) \quad (19)$$

In practice, the effect of the interference generated by collision is stronger than the effect of the additive Gaussian noise, then $\frac{E_s}{d_1^{\mu}} \gg W_0 + W_1$. Hence, eq.(19) can be simplified as

$$P\Big((\frac{d_0}{d_n})^{\mu}10^{\frac{y\epsilon}{10}} > \frac{1}{\rho}\Big) \quad (20)$$

When a collision takes place between subcarriers of two non-transparent relay stations, RS_0 and RS_n, the probability of symbol loss can be derived as follows. Let $A = \frac{d_0}{d_n}$ and $B = 10^{\frac{y\epsilon}{10\mu}}$ be independent random variables, (20) can be simplified as

$$p_{0n} = P(AB > \frac{1}{\rho^{\frac{1}{\mu}}}) = \int_0^A \int_{\frac{1}{\rho^{\frac{1}{\mu}} b}}^{\infty} f_B(b) f_A(a) db da \quad (21)$$

The inner integral can be expressed as

$$\int_{\frac{1}{\rho^{\frac{1}{\mu}} a}}^{\infty} f_B(b) db = P\Big(\epsilon > \frac{-10 ln(\rho a^{\mu})}{y ln(10)}\Big) \quad (22)$$

where $\epsilon = \epsilon_0 - \epsilon_n$ is the sum of two independent Gaussian random variables with zero mean and variance ς^2, and thus $\epsilon \sim N(0, \sqrt{2}\varsigma)$. Then

$$\int_{\frac{1}{\rho^{\frac{1}{\mu}} a}}^{\infty} f_B(b) db = \frac{1}{2} erfc\Big(\frac{-5 ln(\rho a^{\mu})}{\varsigma y ln(10)}\Big) \quad (23)$$

Substituting (23) into (21) gives

$$p_{0n} = \int_0^A \frac{1}{2} erfc\Big(\frac{-5 ln(\rho a^{\mu})}{\varsigma y ln(10)}\Big) f_A(a) da \quad (24)$$

Hence, the probability of symbol loss in a closed form can be expressed as

$$p_{0n} = E_A\Big[\frac{1}{2} erfc\Big(\frac{-5 ln(\rho A^{\mu})}{\varsigma y ln(10)}\Big)\Big] \quad (25)$$

where $erfc(.)$ is the complementary error function of a standard normal distribution and A is the random variable representing the ratio, in each position of RS_0, of the distances to the RS_0 and RS_n ($A = \frac{d_0}{d_n}$). The expectation $E[.]$ is taken over the relay station surface. The proportion of symbols with degraded SNR is given by

$$P(S) = \Big[\frac{M \sum_{n=1}^{r-1} p_{0,n} S_n}{\sum_{n=1}^{n-1} S_n}\Big] \quad (26)$$

where $M = \frac{E_S[g]}{S_0 G}$. Recall that not all symbols involved in a collision are lost, but collision can increase the level of interference making a symbol loss probability high. Therefore, the proportion of symbol with degraded SNR as a function of the number of occupied subchannels can be obtained by (26).

VI. Numerical Results

We consider a multi-hop relay system, each cell is serviced by a BS and six RSs deployed in the coverage area of the BS as presented in Fig.1. Non-transparent relay station mode is used in this scenario, in which the RS operates in different frequency band than the BS. In [1], the usage of the Partially Used Sub-Channeling (PUSC) scheme is specified. In PUSC, all available subcarriers can be assigned in each RS in a centralized manner, therefore no interference between subcarriers in the same RS. The interference is possible between subcarriers simultaneously used in different RSs.
We consider an OFDMA system with $N_c = 1024$ subcarriers, in which there are 120 pilot subcarriers and 184 guard subcarriers. The remaining subcarriers are 720 data subcarriers [1], [6]. In each RS the data subcarriers are grouped into $G = 40$ groups of 18 subcarriers each. A subchannel, which is the smallest assigned resource allocation is achieved by randomly picking a subcarrier from each group. Also, we consider two classes of traffic, class-1 (voice) and class-2 (data). These classes have a number of calls with the same arrival rate following a Poisson Process. The calls belonging to class-1 are assigned one subchannel each, and the service time has a mean $1/\mu_n$ = 2 minutes. On the other hand, calls belonging to class-2 are assigned two subchannels, and for comparison, we consider file that needs mean service time, $1/\mu_n$ = 2 minutes. This corresponds to a file with mean size of 4.6 Mbyte [2]. In our calculations, we set $y = 1/\sqrt{(2)}$ and $\mu = 4$. Table I lists the parameters used to obtain the results in this paper.

TABLE I

Parameter	Value
Number of Base Stations	1
Base Stations Radius (meter)	1000
Number of Relay Stations	6
Relay Station Radius (meter)	350
Log-normal Shadowing (dB)	8

To study the analysis of the system when collision take place between subcarriers in different non-transparent relay stations, we first consider one service class (voice) for which the results are presented in Fig. 2, Fig.3, and Fig.4. Then we continued with our analysis when two service classes are exist (voice and data) for which the results are shown in Fig.5.

Fig. 2 presents the collision rate in the system. When the system becomes more loaded, the chance for MSs in different RSs to use the same subcarriers increases, therefore the SNR degraded frequently as a result of subcarriers collision. From Fig. 2, one can observe that in the case of six interfering RSs, when the load is 22 calls per minute, the collision rate is $\approx$ 0.25, while in the case of two interfering RSs, and the same number of calls, the collision rate is $\approx$ 0.11. Hence, as the system becomes more loaded, and the number of interfering relay stations increases, the collision rate increases.

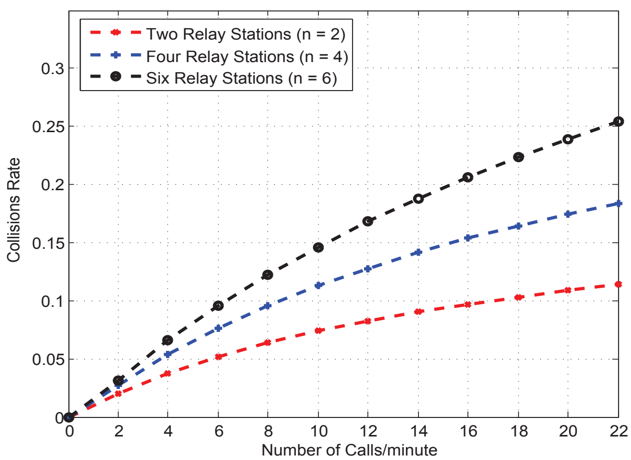

Fig. 2. Collision Rate

Fig. 3 demonstrates the proportion of symbols with degraded SNR when different non-transparent relay stations interfere with each other as a function of call arrival rate. We can observe that when the system becomes more loaded, and the number of interfering RSs increases, the SNR degradation becomes more rapid. This is because more collisions occur in the system. For example, in the case of six interfering RSs, when the load reaches 22 calls per minute, the proportion of symbol with degraded SNR is $\approx$ 0.08, while in the case of two interfering RSs, the proportion of symbol with degraded SNR is $\approx$ 0.045. Thus, as the load increases in the system, more collisions will occur, and more symbols will be lost.

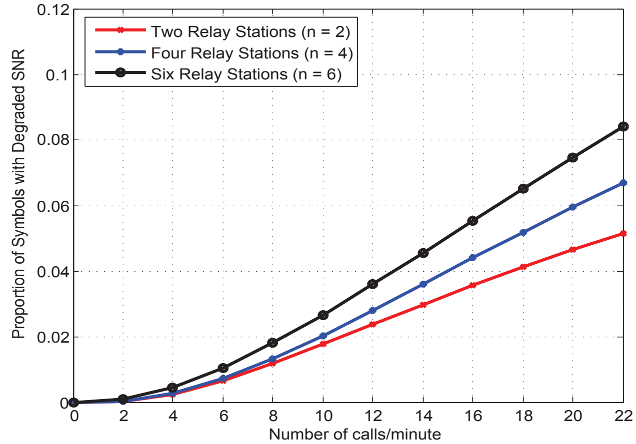

Fig. 3. Proportion of symbols with degraded SNR values

Fig. 4 dipects the symbol loss probability as a function of the required SNR for different interfering non-transparent relay stations. We can observe that at different target SNR values, as the number of interfering RSs increases, the symbol loss probability also increases. For example, in the case of six RSs, when the target SNR is 30 dB, the probability of symbol loss is $\approx$ 0.19, while in the case of two interfering RSs, the probability of symbol loss is $\approx$ 0.06. Also, in this figure, the average symbol loss probability is presented. It can be noted that the average symbol loss probability is $\approx$ 0.08 when the target SNR is 30 dB. Hence, we conclude from this results that the average symbol loss probability due to collisions between subcassiers increases as the number of calls arrive to interfering RSs increases.

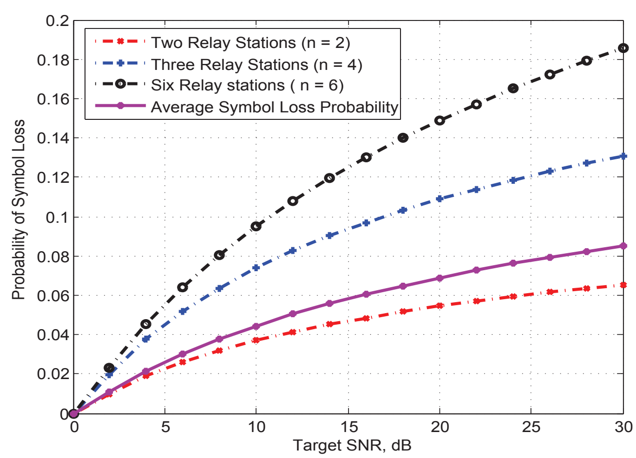

Fig. 4. Probability of symbol loss when collision occurs for different SNR values

Fig. 5 presents the average proportion of symbol with degraded SNR when different types of traffic exist in the system. We can observe that when the network becomes more loaded, the SNR degradation becomes more frequent. As expected the average proportion of symbol with degraded SNR is less in the case of class-1 comparing with the case of class-2. This lower SNR degradation is because class-1 assigned less subcahnnels, therefore the chance for collision to occur is less. For example, when there are 22 calls per minute present in the system , the average proportion of symbol with degraded SNR is ≈ 0.07, while it is ≈ 0.1 in the case of class-2 with the same load (22 calls/min). Hence, we conclude that as more subchannels are occupied as the collision rate increase, consequently the proportion of symbol with degraded SNR increase resulting in the symbol to be lost.

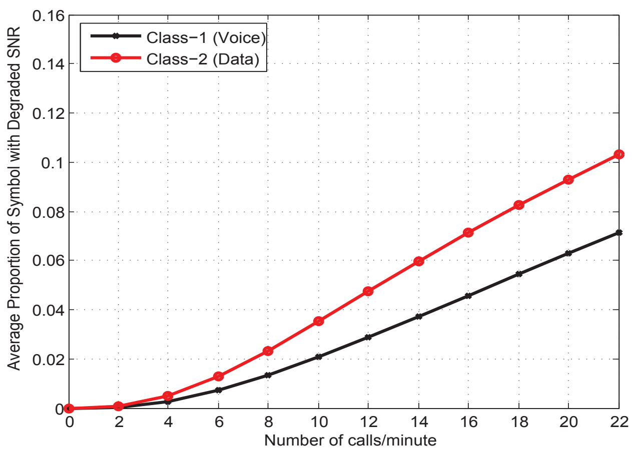

Fig. 5. Average Proportion of symbols with degraded SNR values for class-1 and class-2

VII. Conclusion

In this paper, we developed an analytical model to obtain the expected number of collisions of OFDMA technique in mobile multi-hop relaying system. In our analytical model, we calculated the expected number of collisions resulting from the simultaneous use of subcarriers in different non-transparent relay stations. Then for different numbers of interfering RSs, we obtained the symbol loss probability as a function of the required SNR as well as we calculated the average symbol loss probability in the system. In addition, we obtained the proportion of symbol with degraded SNR, moreover the average proportion of symbol with degraded SNR when two service classes (voice and data) are exist is calculated and we showed that the SNR degradation becomes more frequent as more calls arrive to the system. Finally, we showed that the collision rate increases as the number of calls increases.

Acknowledgment

This work is supported by a grant from the Natural Sciences and Engineering Research Council (NSERC) of Canada. The first author would like to thank the Ministry of Higher Education in Libya for the financial support.

References

[1] IEEE Std 802.16j-2009, "IEEE Standard for Multihop Relay networks, Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems, Amendment for Physical and Medium Access Layers for Mobile multihop Relay," June. 2009.

[2] IEEE Std 802.16e-2005, "IEEE Standard for Local and metropolitan area networks, Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems, Amendment for Physical and Medium Access Layers for Combined Fixed and Mobile Operation in Licensed Bands," Feb. 2006.

[3] IEEE C802.16e-04/453r2,"Add sub-segment to the PUSC mode," *Huawei*, Nov. 2004.

[4] C. Sankaran, F. Wang, and A. Ghosh"Performance of Frequency Selective Scheduling and Fractional Frequency Reuse schemes for WIMAX," *IEEE VTC-Spring*, pp. 1-5, April, 2009.

[5] C. Tarhini, and T. Chahed,"On capacity of OFDMA-based IEEE802.16 WiMAX including Adaptive Modulation and Coding (AMC) and inter-cell interference," *IEEE LANMAN*, pp. 139-144, June, 2007.

[6] J. Andrews, A. Ghosh, and R. Muhamed "Fundamentals of WiMAX," *Book*, pp. 462 - 472, 2007.

[7] T. Kurt, and H. Delic,"On Symbol Collisions in FH-OFDMA," *IEEE VTC-Spring*, pp. 1859-1863, May, 2004.

[8] H. Ahmed, A. Sulyman,H. Hassanein, "BER Performance of OFDM Systems with Channel Impairments," *IEEE LCN*, pp. 1027 - 1032, Oct., 2009.

[9] A. Radwan, and H. Hassanein,"Capacity Enhancment in CDMA Cellular Networks Using Multi-hop Communication," *IEEE ISCC*, pp. 139-144, June, 2006.

[10] S. Ghahramani, "Fundamentals of Probability," *Book*, second Edition, 2000.